

Designing Bank Warning System Based on Logit Model Fitted to Multi-Period Performance Data

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Introduction

This paper is concerned with designing a bank warning system based on readily available performance data, which would identify weak banks. The warning system would allow regulatory agencies to intervene in the weak banks on a timely basis and avoid some of the bank failures, or at least reduce the consequent losses resulting from such failures [1,2,3,4].

The current system followed by the regulatory agencies to identify 'problem banks' is based on on-site examination reports [7,8]. We strongly feel that the identification of problem banks should be largely based on reports routinely submitted by banks and not on examination reports. It is important that the limited available resources of the regulatory agencies for on-site examination are used most effectively. An important objective of the proposed warning system is to classify banks into safe, weak and very weak categories and base the frequency of on-site examinations on this risk classification.

It is reasonable to assume that any deterioration of the financial conditions of a bank occurs gradually, excepting in situations where fraud or embezzlement is involved. It should be feasible to capture this deterioration from periodic performance reports. The present system of generating a list of problem banks from on-site examination reports is not correct. The bank failures should, of course, be prevented through a good design of the banking sector. However, it is perhaps not possible to come up with a design which would completely eliminate bank failures. There would always be a need for intervention and dynamic control. We need timely intervention before financial difficulties of a bank become real serious.

A warning system would incur two types of errors. A type I error is incurred when a bank labelled safe is later found to be unsafe. A type II error is incurred when a bank labelled unsafe (weak or very weak) is later found to be safe. The cost of type I error is much higher than the cost of type II error. In the decision procedure for classifying banks into weak and safe banks, type I error is kept very low. Consequently, the level of type II error is high. An erroneous concern to keep type II error low has been responsible for the current procedure of identifying problem banks on on-site examination reports. The lead time available in the current procedure for useful intervention is small.

In an ideal control setup of a bank, its stockholders should monitor its financial condition, and there should be no need for intervention. Stockholders can be expected to exercise adequate controls if they have an adequate equity stake. Capital adequacy is an important design parameter. A bank at start might have had adequate equity, but investment losses on bad loans over time might have eroded its equity, leaving a need for the regulatory agencies to watch interest of the depositors.

A bank gradually moving into weak and very weak categories requires models somewhat different from traditional survival models. In survival models, one

is concerned with determining how, with the passage of time, the remaining life decreases as a ratio of the remaining life. This ratio is usually known as 'hazard rate'. In the traditional survival model studies, hazard rate is expressed as a function of time t . In the case of banks' warning system, time needs to be replaced by some other appropriate composite non-performance measure, and hazard rate should be expressed as a monotonic function of this measure. The quality of the warning system critically depends upon:

- (i) Identifying the vector of significant explanatory variables x ;
- (ii) Integrating information contained in these variables into a composite scalar quantity.

The two methods used for integrating step (ii) are:

Method 1 Integrates various explanatory variables linearly with weighting vector β_1 , into a composite non-performance scalar quantity $w = \beta_1'x_1$, and defines probability of failure h as a monotonic increasing function of w .

Method 2 Defines w non-linearly as follows: $w = \beta'x_1 + x_1'Bx_1 = \beta_2'x_2$, and defines h as a monotonic increasing function of w . Method 2 includes significant square of variable terms, and cross-products of variables terms in pairs.

The data is analyzed by both methods, and the results are reported in the section on Results. It may be pointed out that the warning system design problem is quite different from the banking sector design problem. In the former problem, the objective is to define a risk classification scheme for banks on readily available performance records so that frequency of on-site inspections can be based on this risk scheme, whereas in the later problem, the objective is to find a design for the banking sector which would minimize the probability of failure for a bank, in general.

In a two categories risk classification scheme, the warning system essentially tests the following hypothesis for each bank i

$$H_0: h_{it, \tau} \geq h^0 \quad ; \quad H_a: h_{it, \tau} < h^0 \quad \dots (1)$$

where h^0 is a pre-specified critical hazard rate, which becomes the basis for classifying all banks into two categories--risky and non-risky. The desired value α for type I error is assumed to be given.

In a three way classification scheme into very risky, risky, and safe banks, the testing of hypothesis problem may be modified accordingly:

$$\begin{aligned} H_0: h_{it, \tau} &\geq h^1 \\ H_1: h^0 &\leq h_{it, \tau} < h^1 \\ H_3: h_{it, \tau} &< h^0 \end{aligned} \quad \dots (2)$$

The associated α_1 and α_2 values for misclassification of the risky and very risky classes into safe category are assumed to be given.

We may re-emphasize that an important objective of the warning system is to give a warning signal as quickly as possible when a safe bank transitions into a risky category or a risky bank transitions into a higher risk category. A bank, once labelled as belonging to 'risky or very risky' category, is going to stay with this label until once again it is re-admitted

into the safe category. Banks in risky categories are going to be given more frequent on-site inspections.

The important categories of performance variables which have been considered in the risk classification scheme of a bank are: Quality of assets; Capital adequacy; and Management. The quality of assets, particularly, the quality of loans are very important in influencing how risky a bank is. The book value accounting system does not easily reveal financial weakness, but several important performance variables conjunctively do contain this information, and the challenge lies in identifying this information.

Sample Selection Criterion

Two samples are used in the study, one is called the learning sample, and second the testing sample. The data in the learning sample are used to identify the subset of important explanatory variables $\{x\}$, and to estimate the coefficients vector β_1 in method 1 and β_2 in method 2, which transform the multiple performance measure into a composite scalar non-performance measure w . The observed value of the dependent variable y for a bank is binary in nature. It assumes the value of 0 or 1 for a period, depending upon whether the bank had not failed, or failed respectively. In the proposed model, the failed state is assumed to be an absorbing state, implying that once a bank has failed, it does not return to a non-failed state. Since failure state has been assumed to be an absorbing state, a '1' cannot precede a '0'.

The learning sample included 194 failed banks and 100 non-failed banks. The failed banks had failed during 1990 and the first half of 1991 [6]. The problem results into a sample of 650 observations, 194 pertain to failed banks and 456 pertain to non-failed banks. In future discussion in this paper, 650 observations refer to 650 banks. The non-failed banks were selected on stratified random basis, the stratification being based on FDIC designated regions, states within regions, asset size and SMSA/non-SMSA areas from the list of non-failed banks. Each observation in the learning sample consisted of 25 performance variables. These variables are grouped into following categories:

Size & Growth:	How fast the bank has been growing? Asset Size Growth: Assets percent change during 12 months
Loan exposure:	Is the bank's loans heavy in the higher risk areas? Construction Loan/Total Assets Commercial Real Estate/Total Assets Consumer Loan/Total Assets Agricultural Loan/Total Assets
Capital Adequacy:	How strong is the bank's capital position? Core Capital/Total Assets (Core Capital+Loan Loss Reserves)/Total Assets Nonperforming Assets/(Core Capital+Loan Loss Reserves)
Asset Quality:	What are the amounts and trends in non-performing loans? Nonperforming Assets/Total Assets 90+days past due loans/Gross Loans Nonaccrual Loans/Gross Loans Net YTD chargeoffs/Average Loans

Earnings: What are the sources of earnings and how does overall profitability compare to other banks?
Interest Income/Average Assets
Interest Expense/Average Assets
Net Interest Income/Average Assets
Noninterest Income/Average Assets
Overhead Expense/Average Assets
Provision for Loan Losses/Average Assets
Securities Gains(Losses)/Average Assets
Return on Average Assets
Return on Average Equity
Income Before Extraordinary Items

Liquidity: What portion of the bank's assets are readily convertible to cash?
Liquid Assets/Total Liabilities
\$100,000 & Over Time Deposits/Total Assets

The forward regression variables selection method was used, which utilizes likelihood ratio test. The following 11 variables were found to be significant in the linear logit model. In the quadratic logit model, the 11 significant explanatory variables in method 1, their square terms, and some cross products terms of the explanatory variables which had high correlations were used. The estimates of the coefficients β_1 and β_2 for the linear and quadratic logit models respectively are shown in Figure 1 [5].

Category	Variables	$\hat{\beta}_1$	$\hat{\beta}_2$
1. Size and Growth	Smaller Asset Size (Smaller Asset Size) ²	.00034	.000269 0.0
2. Capital Adequacy	Smaller Core Capital/Total Assets (Smaller Core Capital/Total Assets) ² (Smaller Core Capital/Total Assets) x (Return on Average Assets/Average Assets) (Smaller Core Capital/Total Assets) x (Interest Expense/Average Assets)	.5324	.1768 .00154 .0246 .1553
3. Asset Quality	Nonperforming Assets/Total Assets 90 plus days past due loans/Gross loans (Nonperforming Assets/Total Assets) ² (90+ Days past due loans/Gross Loans) ² (Nonperforming Assets/Total Assets) x (Return on Average Assets/Average Assets) (Nonperforming Assets /Total Assets) x (90 plus days past due loans/Gross Loans)	.1462 .0176	.2280 .00576 .00014 .00151 .0284 .00331
4. Earnings	Decreasing Return on Average Assets /Average Assets Overhead Expense/Average Assets Decreasing Year-todate Income Increasing Interest Expense /Average Assets Return on Average Equity (Decreasing Return on Average Assets /Average Assets) ²	.1433 .2774 .0112 .0802 .00653	.1933 .0364 .0193 .4063 .00466 .00185

Figure 1: Estimates of the coefficients β_1 and β_2 for the linear and quadratic logit models for methods 1 and 2

Category	Variables	$\hat{\beta}_1$	$\hat{\beta}_2$
4. Earnings (continued)	(Overhead Expense/Average Assets) ²		.0219
	(Decreasing Year-to-date Income) ²		.00001
	(Increasing Interest Expense/Average Assets) ²		.0221
	(Return on Average Equity) ²		.00005
	(Decreasing Return on Average Assets /Average Assets) x		.00145
5. Liquidity	(Overhead Expense/Average Assets)		
	\$100,000 and over Time Deposits/Total Assets	.0433	.0721
	Liquid Assets/Total Liabilities	.0158	.0225
	(\$100,000 and over Time Deposits/Total Assets) ²		.00076
	(Liquid Assets /Total Liabilities) ²		.000094
	(\$100,000 and over Time Deposits /Total Assets) x		.00027
	(Liquid Assets/Total Liabilities)		

Figure 1: Estimates of the coefficients β_1 and β_2 for the linear and quadratic logit models for methods 1 and 2
(continued)

The test sample is used to test how well our model (which has learnt from the learning sample) performs. The test sample does not include banks which were included in the learning sample. The test sample comprised of performance data for 60 banks, 30 banks that had failed in second half of 1991 and 30 non-failed banks. The performance predicted by the model is compared with actual performance for the banks in the test sample in the section on Results.

Mathematical Model

Time is assumed to be in discrete units. Let τ be a six-months period. The index τ is subduced initially. Recall that y_{it} denotes the binary dependent variable for bank i , assuming value of '0' or '1', where '0' implying that bank did not fail and '1' implying that the bank failed during t to $t+\tau$. x_{it} denotes the vector of explanatory performance variables of bank i .

The hazard function h_{it} was given by

$$h_{it} = \text{Prob}[y_{it} = 1] = F(w_{it}) \quad \dots (3)$$

It is desired that the hazard function h_{it} is expressed as a monotonic increasing function of w_{it} . h_{it} should also vary within the range 0 to 1. There are a great many functions which would meet these requirements. Logit function is one of them, and it has been used in our model. Logit function has well developed theory and is well supported with available software packages [5]. According to the logit function, value of h_{it} is given by

$$h_{it} = \text{Prob}[y_{it}=1|w_{it}] = e^{w_{it}} / (1+e^{w_{it}}) \quad \dots (4)$$

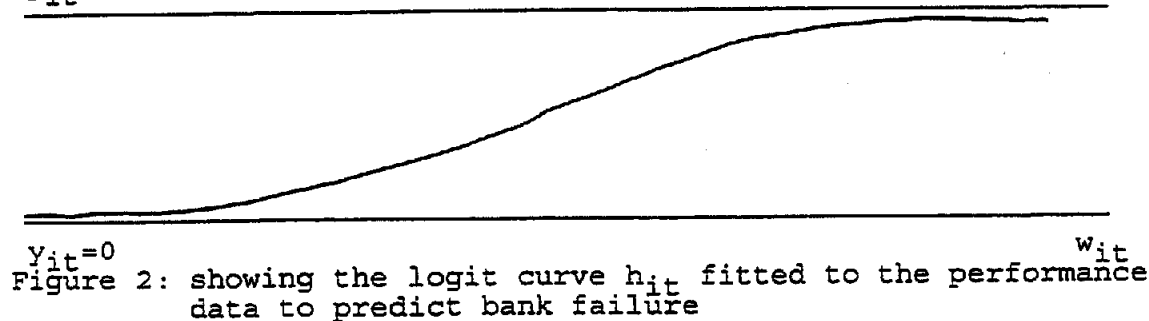
$$1 - h_{it} = \text{Prob}[y_{it}=0|w_{it}] = 1 - e^{w_{it}} / (1+e^{w_{it}}) = 1 / (1+e^{w_{it}})$$

It may be easily verified that $h_{it} \rightarrow 1$, as $w_{it} \rightarrow \infty$ and $h_{it} \rightarrow 0$, as $w_{it} \rightarrow -\infty$. w_{it} is a composite non-performance measure.

Let T denote the study time for which performance data is available for the learning sample. T is an integer multiple of τ . For a bank which did not fail, we will have T values for the dependent variable y_{it} , each equal to '0'.

However, for a bank which failed, we may have values fewer than T values, depending upon when the bank failed. Let T_i denote the number of performance values available for bank i which failed. The last value of the dependent variable for failed bank will equal 1, and all the values preceding 1 will be '0's.

$Y_{it}=1$



The observed values of the dependent variable for the learning sample fall on the two parallel lines shown in Figure 2. The coefficient vectors β_1 and β_2 in the logit functions for models 1 and 2 respectively are computed by maximizing the respective likelihood functions for the two models of the observed values for the learning sample.

The likelihood of the data may be written as

$$L = \prod_{i \in S_0} \prod_{t=1}^T (1 - h_{it}) \prod_{i \in S_1} \prod_{t=1}^{T_i-1} (1 - h_{it}) h_{iT_i}$$

$$= \prod_{i \in S_0} \prod_{t=1}^T \{1/(1+e^{\beta'x_{it}})\} \prod_{i \in S_1} \prod_{t=1}^{T_i-1} \{1/(1+e^{\beta'x_{it}})\} \{e^{\beta'x_{iT_i}}/(1+e^{\beta'x_{iT_i}})\}$$

where S_0 and S_1 denote the sets of banks in the learning sample which did not fail, and which failed respectively.

The conversion of each observation vector into a composite scalar, and ordering of the scalars in Figure 1 is achieved during the process of maximizing the likelihood function. Since h is a monotonic increasing function of $\beta'x$, we can assume that the order of not allowing a 0 to succeed a 1 for y values is automatically complied through optimization, without our being explicitly concerned about it. Hence, maximizing the likelihood with multiple reading for a bank is equivalent to having one reading for multiple banks [9]. The likelihood function may be re-written with another index i' , where i' ranges over sets S_0' and S_1' . The number of 0's and 1's in S_0' and S_1' are equal to the total number of 0's and 1's in sets S_0 and S_1 over multiple period respectively. Likelihood function may be re-written as (β could be β_1 or β_2)

$$L = \prod_{i' \in S_0'} \{1/(1+e^{\beta'x_{i'}})\} \prod_{i' \in S_1'} \{e^{\beta'x_{i'}}/(1+e^{\beta'x_{i'}})\}$$

$$= \prod_{i' \in S'} \{1/(1+e^{\beta'x_{i'}})\}^{1-Y_{i'}} \{e^{\beta'x_{i'}}/(1+e^{\beta'x_{i'}})\}^{Y_{i'}}$$

where S' is the union of sets S_0' and S_1'

$$= \prod_{i' \in S'} (1 - h_{i'})^{1-Y_{i'}} (h_{i'})^{Y_{i'}} \quad \dots (5)$$

$$\log L = - \sum_{i' \in S'} (1-y_{i'}) \log(1+e^{\beta' x_{i'}}) + \beta' \sum_{i' \in S'} x_{i'} y_{i'}$$

$$- \sum_{i' \in S'} y_{i'} \log(1+e^{\beta' x_{i'}})$$

$$\log L = \sum_{i' \in S'} [(1-y_{i'}) \log(1-h_{i'}(\beta' x_{i'})) + y_{i'} \ln(\beta' x_{i'})] \quad \dots (6)$$

To find maximum likelihood (ML) estimate of β , differentiating $\ln L$ with respect to β yields

$$S(\beta) = \frac{\partial \log L}{\partial \beta} = \frac{\sum_{i' \in S'} h_{i'}(\beta' x_{i'}) x_{i'} \{1 - y_{i'} h_{i'}(\beta' x_{i'})\}}{[1-h_{i'}(\beta' x_{i'})] h_{i'}(\beta' x_{i'})} = 0 \quad \dots (7)$$

Newton Raphson Method is used to solve equation (7). The information matrix is

$$I(\beta) = E \left(- \frac{\partial^2 \log L}{\partial \beta \partial \beta'} \right) \quad \dots (8)$$

Starting with some initial value of β , say β_0 , we compute the values $S(\beta_0)$ and $I(\beta_0)$. Then the new estimate of β is obtained, by the method of scoring,

$$\beta_1 = \beta_0 + [I(\beta_0)]^{-1} S(\beta_0) \quad \dots (9)$$

This iterative procedure is repeated till convergence is attained. After β has been estimated, we can compute h_{it} as a function of $\beta' x_{it}$ given by (4). h_{it} gives an estimate of the probability that bank i would fail over the duration t to $t+\tau$, where τ in our case represents next 6 months.

Empirical Performance Model

We consider a two-way risk classification. The extension to the three-way risk classification scheme can be easily done. An important rule for choosing appropriate null and alternate hypothesis, H_0 and H_a , is that we choose the hypothesis we wish to reject, as H_0 . We establish H_a by rejecting H_0 . As stated earlier, type-I error in our problem would occur, when a bank labelled safe by the warning system is found unsafe later on. Type-II error occurs when a bank labelled unsafe is found to be safe, as shown in Figure 2. The cost of type-I error is much higher than the cost of type-II error. Type-I error is controlled through choosing an appropriate value of α , and α is usually kept at a low value.

		Our verdict	
		H_0 Bank is labelled unsafe	H_a Bank is labelled safe
H_0 The bank does fail Nature	H_0 The bank does fail		Type-I error
	H_a The bank doesnot fail	Type-II error	

Figure 2: showing what constitutes type I and type II errors when initiating two way classification of banks

To achieve a desired classification, we determine w^0 , a value of $\beta_1'x_1$, which discriminates between the healthy and the weak banks. The above concept of separating weak banks from healthy banks may be described in the following way and also illustrated in Figure 3. Let A be the set of all banks. Let B be the set of problem banks. Let C be the set of banks which are identified as weak banks by the above scheme. An ideal search will be, when set C completely contains set B. However, since randomness is involved, set C may not completely contain set B. Portion of set B which is not covered by set C, that is, $(B-B \cap C)$, corresponds to type-I error. Similarly, portion $(C-B \cap C)$ of set C which is not covered by set B, corresponds to type-II error. The challenge lies in generating set C, as small as possible, which would have α within specified limits.

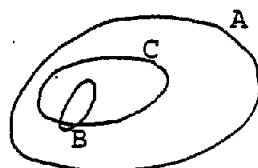


Figure 3: Venn diagram showing set of all banks, A; set of weak banks, C; and the set of problem banks, B.

Results

The truncation point $h^0 = h(w^0) = .1$ is used in this study. The corresponding value of w^0 may be achieved by solving $w^0 = \ln h^0 / (1 - \ln h^0) = -2.197$. The results for the linear logit model for the learning sample are given in Figure 4 (The model's estimators were obtained from the learning sample). Type-I error is about 4% while the type-II error is 30%.

Our Analysis Result

	H_0 Bank is labelled unsafe	H_a Bank is labelled safe
H_0 The bank does fail	187 (.96)	7 (.04)
Real Data		
H_a The bank doesnot fail	127 (.30)	329 (.70)

Figure 4: showing classification for the learning sample for the linear logit model

Recall, the test sample contained 60 banks, 30 failed and 30 non-failed banks. For the truncation point $h^0 = h(w^0) = .1$, classification for the test sample is shown in Figure 5.

Our Analysis Result

	H_0 Bank is labelled unsafe	H_a Bank is labelled safe
H_0 The bank does fail	28 (.93)	2 (.067)
Real Data		
H_a The bank doesnot fail	0 (0)	30 (1.0)

Figure 5: showing classification for the test sample using the model estimators from the learning sample for the linear logit model

For the quadratic logit model, the probability of failure, h is a monotonic increasing function of $w = \beta_2'x_2$, and the truncation point $h^0 = h(w^0) = .1$ may be equivalently achieved through $w^0 = \ln h^0 / (1 - \ln h^0) = -2.197$. The performance of the quadratic logit model for the learning sample are given in Figure 6. Type-I error is about 4.6% while the type-II error is 24.8%.

Our Analysis Result

	H_0 Bank is labelled unsafe	H_a Bank is labelled safe
H_0 The bank does fail Real Data	185 (.954)	9 (.046)
H_a The bank doesnot fail	113 (.248)	343 (.752)

Figure 6: showing classification for the learning sample for the quadratic logit model

Performance of quadratic logit model for the test sample of 30 failed and 30 non-failed banks with the truncation point $h^0 = h(w^0) = .1$, is shown in Figure 7.

Our Analysis Result

	H_0 Bank is labelled unsafe	H_a Bank is labelled safe
H_0 The bank does fail Real Data	28 (.93)	2 (.067)
H_a The bank doesnot fail	0 (0)	30 (1.0)

Figure 7: showing classification for the test sample using the model estimators from the learning sample for the quadratic logit model

Conclusions

This paper was aimed at developing a risk classification scheme for banks, based on performance data which are readily available to the regulatory agencies in periodic call reports submitted to them. The objective is to develop an on-site examination scheme with varying frequencies. An important variable in the determination of on-site examination frequency is risk category of a bank. Of-course, size of the bank is another important variable. This paper does not go into determining optimal frequencies.

The learning sample included 194 banks that had failed during 1990 and first half of 1991, and 456 banks that did not fail. The non-failed banks were picked on a stratified random sampling basis. Performance for the learning sample, and test sample are found to be quite satisfactory.

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