

Using SAS Software to Design and Evaluate Military Standards Sampling Plans

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ABSTRACT

Although the trend in modern quality control is a move towards on-line statistical process control through the use of control charts and control algorithms, lot acceptance sampling is still heavily used in industry particularly by governmental suppliers. Lot acceptance sampling consists of sampling a lot of finished product and determining whether the lot is of acceptable quality based on some quality characteristic. Two of the most used sampling plans for lot acceptance sampling are Military Standards 105 and 414. In this presentation, SAS code sequences and examples will be described that show how to use the SAS data step along with functions available in SAS/QC® to automate the design of sampling plans shown in these military standards.

1. Introduction

Although acceptance sampling is thought by some quality practitioners to be an outdated method for quality improvement, the technique is still widely used in industry today. In this paper, two of the most widely used sampling plans, US Department of Defense Military Standard 105 (1963) and Military Standard 414 (1957), will be discussed. It will be seen that with the help of SAS software a much broader range of sampling plans than is shown in these military standards can be designed and evaluated.

The basis for acceptance sampling is quite simple. A group of homogeneous product called a lot is produced by a supplier. In order to determine if the quality of the manufactured goods is acceptable to the consumer, a sample of product is taken from the lot and a quality characteristic is measured on each item in the sample. If the quality of the sample is at an acceptable level then the lot is accepted by the consumer. If the quality of the sample is unacceptable then the lot is rejected and returned to the supplier.

There are two basic types of acceptance sampling plans, attribute and variable. In attribute sampling plans, the quality characteristic measured on each item is dichotomous (pass/fail, nondefective/defective, etc.). With variable sampling plans, the quality characteristic measured on each item is continuous (length, width, temperature, etc.). Military Standard 105 contains attribute sampling plans while Military Standard 414 contains variable sampling plans.

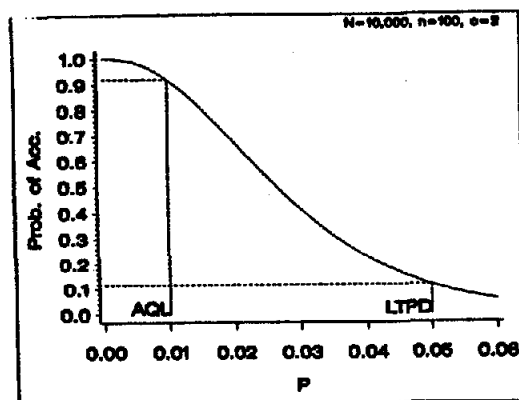
When a quality practitioner is considering implementing an attribute or variable sampling plan, his/her decision should be based on the probability of passing lots of "good" quality, the probability of passing lots of "bad" quality and the amount of sampling necessary to conduct the plan. "Good" quality lots are defined to be those whose proportion of defectives are at an acceptable quality level (AQL). "Bad" lots are defined as those whose proportion of defectives are at a rejectable quality level. This level is sometimes called the lot tolerance percent defective (LTPD).

The probability of acceptance, P_a , for a sampling plan is the probability of accepting a lot given that the stream of lots coming through the inspection process has a proportion of items defective p . For a lot size N containing $k=Np$ defectives P_a is equal to the probability of finding c or less defectives in a sample of size n . The correct probability model for this situation is the hypergeometric model,

$$P_a = P(x \leq c) = \sum_{x=0}^c \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}} \quad x = 0, 1, 2, \dots, \min(n, k)$$

which can be computed using the PROBHYPR function in base SAS software. To review the concepts just discussed, consider the operating characteristic (OC) curve shown in Figure 1. The OC curve is a plot of the probability of passing a lot over a range of lot percent defective values.

Figure 1. Sample OC Curve



From Figure 1, it can be seen that the probability of acceptance at AQL=.01 is $\approx .92$ while the probability of acceptance at LTPD is $\approx .12$. These two values, P_a at LTD and P_a at LTPD, are the basis for constructing most if not all sampling plans used in industry.

2. Military Standard 105 Plans

Military Standard 105 has long been used to design sampling plans for quality characteristics measured as conforming/non conforming (attribute data). The plans shown in this standard are based on the probability of passing a lot at AQL. Although the plans involve various switching rules for tightened and normal inspection, the tables and graphs shown in the standard are based on AQL at normal inspection levels.

The OC curves of any of the plans in Military Standard 105 can be verified using SAS software. For example, with $N = 2,000$, the use of sample size code letter K is indicated from Table I of the Standard. From Table II-a of the Standard $n = 125$ and $c = 3$ for AQL = .01. To verify the values of P_a for this plan, use the following SAS code:

```
data oc105;
  n=125; c=3; lotn=2000;
  do p=.00658, .01, .0109, .014, .0203, .0294, .0409, .062, .0804;
    k=ceil(lotn*p);
    pa=probhypr(lotn,k,n,c);
    output;
  end;
run;
```

```
proc print;
title 'OC Curve Values for MIL-STD-105 Plan';
title2 'N=2,000, n=125, c=3';
run;
```

As can be seen from Output 1 in the appendix, the probability of acceptance at AQL=.01 is 0.96775. The probability of accepting a lot containing 6.2% defectives with this plan is 0.04055. These values correspond to Table X-K-1 in the Standard. Any plan in Military Standard 105 can be evaluated in this way.

With the SAS probability functions, we are not limited to those plans given on the Military Standard 105 tables. For instance, to find a plan which guarantees that the probability of acceptance at AQL = 0.8% defective with the acceptance number being $c = 1$ is 90%, use the following SAS code:

```
data newoc;
  lotn=2000; c=1; nn=5; aql=.008;
  do until (paa<.90);
    n=nn; pa=paa;
    k=ceil(aql*lotn);
    paa=probypr(lotn,k,nn,c);
    nn=nn+1;
  end;
drop nn paa;
run;

proc print;
title 'Sample Size to Achieve Specified PA at AQL';
title2 'N=2,000, c=1';
run;
```

From Output 2 in the appendix, the sample size necessary to meet the criteria given above is $n=68$.

3. Military Standard 414 Plans

Variable sampling plans are used in situations where the quality characteristic upon which the decision to accept or reject a lot of product is continuous in nature. Once again, the SAS data step can be employed to compute the parameters and characteristics of these plans.

As with most variables sampling plans, it is assumed that the quality characteristic being measured follows a normal distribution. For all plans discussed in this paper, it will be assumed that both the mean, μ and standard deviation, σ , of this normal distribution are unknown and must be estimated from the sample data.

Variable sampling plans are conducted in the same manner as attribute plans in that a sample of size n is taken from a lot of size N . Unlike the attribute plans, a continuous response is recorded on each of the sampled units and a mean, $\bar{X}$, and standard deviation, s , are calculated from the sampled data using the formulas $\bar{X} = \sum_{i=1}^n X_i / n$ and $s = \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 / (n - 1)}$. The criteria for accepting or rejecting the lot is then based on these estimated quantities.

The variables sampling plans of Military Standard 414 are, like those of Military Standard 105, based on the probability of accepting a lot at AQL. There are two ways that these plans are implemented. Both can be programmed using the SAS data step.

3.1 The k-Method Procedure

For a one sided lower specification limit, L , for lot quality, calculate $Z_1 = (\bar{X} - L)/s$ and reject the lot if $Z_1 \leq k$. We need to determine the sample size, n , and the cutoff value, k , to guarantee that $P(Z_1 \leq k | p = p_1) = \alpha$ and $P(Z_1 \leq k | p = p_2) = 1 - \beta$

Duncan(1974) has shown that these equations can be approximately satisfied when

$$k = \frac{Z_\alpha Z_{p_2} + Z_\beta Z_{p_1}}{Z_\alpha + Z_\beta}$$

and

$$n = \left(1 + \frac{k^2}{2}\right) \left(\frac{Z_\alpha + Z_\beta}{Z_{p_1} - Z_{p_2}}\right)^2$$

where Z_p is the (upper) p th percentile of the standard normal distribution. This value is given by the SAS function PROBIT. The data step shown below computes k , n to satisfy these two equations.

```
data kmethod;
  alpha=.05; beta=.1; p1=.01; p2=.08;
  zalpha=probit(1-alpha);
  zbeta =probit(1-beta);
  zp1=probit(1-p1);
  zp2=probit(1-p2);
  k=(zalpha*zp2 + zbeta*zp1) / (zalpha + zbeta);
  n=(1 + (k**2/2)) * ((zalpha+zbeta)/(zp2-zp1))**2;
  drop zalpha zbeta zp1 zp2;
run;

proc print;
title 'Variable Sample Plan - K Method Procedure';
run;
```

For a plan with $P_s = .95$ at $AQL = .01$ and $P_s = .10$ at $LTPD = .08$, the code gives $k = 1.809$ and $n = 27$. If the lower specification limit for a lot is 17000 and $\bar{X} = 18526$ with $s = 754$ then $Z_1 = (18526 - 17000)/754 = 2.024 > k = 1.809$ which implies that we accept the lot.

3.2 The M-Method Procedure

A second way that a variables sampling plan is implemented is the M-Method procedure. Once the mean and standard deviation of the distribution of the process characteristics have been estimated, the fraction of nonconforming product, p , can be estimated and the decision to accept or reject the lot can be based on rejecting the lot when p is greater than some predetermined limit M .

Leiberman and Resnikoff(1955) have shown that the minimum variance unbiased estimated (MVUE) fraction nonconforming is

$$p_L = p(\bar{X}, s^2) = \int_0^{\max(0, 5-V)} f(x, a, b) dx \quad (1)$$

where $f(x, a, b)$ is a beta distribution with parameters $a = b = (n/2) - 1$ and $V = (\bar{X} - L)/s(\sqrt{n}/(n-1))$. With n and k known, the value of M can be computed from $M = \int_0^Z f(x, a, b) dx$ where $Z = .5 - (k\sqrt{n}/2(n-1))$. The equations can be programmed by using the SAS function PROBBETA. The code to estimate p_L and find M is shown below.

```

data mmethod;
  xbar=18526; s=754; l=17000; n=27; k=1.809;
  a=(n/2)-1; b=(n/2)-1;
  z1 = max(0, .5-.5*((xbar-l)/s)*(sqrt(n)/(n-1)));
  phat1 = probbeta(z1,a,b);
  z3 = .5 - (k*sqrt(n))/(2*(n-1));
  m = probbeta(z3,a,b);
  drop a b z1 z3;
run;

proc print;
title 'Variables Sampling Plan - M Method - One Sided Spec Limit';
run;

```

From Output 4 in the appendix, $p_L=.0182 < M=.0319$ so we accept the lot. The procedure can easily be modified to use with a single upper specification limit.

3.3 OC Curves for Military Standard 414 Plans

The SAS data step can also be used to check performance characteristics of Military Standard 414 plans. Leiberman and Resnikoff (1955) have shown that the OC curve for the Military Standard 414 plans is given by $P_p = P(T^* < \sqrt{nk})$ where T^* has a noncentral T distribution with $n-1$ degrees of freedom and noncentrality parameter $\sqrt{nk_p}$ where k_p is found from

$$\int_{k_p}^{\infty} f(z) dz = p$$

where $f(z)$ is the pdf for the Normal distribution. This second equation relates the fraction defective to the sampling plan OC curve. Once again, SAS probability functions can be used to compute the necessary quantities.

```

data oc414;
  n=30; k=1.86;
  do p=.01 to .15 by .01;
    kp=probit(1-p);
    nc=sqrt(n)*kp;
    pa=1-probt(sqrt(n)*k,n-1,nc);
    output;
  end;
run;

proc print;
title 'OC Curve Values for MIL-STD-414 Plan';
run;

```

The values of the OC curve for the plan with sample size code letter J and AQL=.01 is shown in Output 5 in the appendix.

3.4 M-Method Procedure - Double Specification Limits

For two sided specification limits, the M-method is the more easily implemented of the two methods discussed in the previous sections. A lot of product is rejected if $p_u + p_l > M$ where p_u and p_l are the estimated proportion defective above the upper specification limit and below the lower specification limit, respectively. The value of p_u can be calculated using (1) with $V=((U-\bar{X})/s)/(\sqrt{n/(n-1)})$. These SAS statements can be used to make the calculations.

```

data mmethod;
  xbar=18526; s=754; L=17000; U=20000; n=27; k=1.809;

```

```

a=(n/2)-1; b=(n/2)-1;
z1 = max(0 , .5-.5*((xbar-L)/s)*(sqrt(n)/(n-1)));
phat1 = probbeta(z1,a,b);
z2 = max(0 , .5-.5*((U-xbar)/s)*(sqrt(n)/(n-1)));
phatu = probbeta(z2,a,b);
z3 = .5 - (k*sqrt(n))/(2*(n-1));
m = probbeta(z3,a,b);
drop a b z1 z2 z3;
run;

proc print;
title 'Variable Sampling Plan - M Method - Two Sided Spec Limits';
run;

```

From Output 6, $p_L + p_U = .0182 + .0220 = .0402 > .0319$ so the lot is rejected when both the upper and lower specification limits are considered.

4. Summary

In this paper, it has been shown that the design of sampling plans like those shown in Military Standards 105 and 414 can be automated using SAS software. The values of the OC curves for the plans in the standards can also be generated using the SAS data step.

References

- Duncan, A.J. (1974). *Quality Control and Industrial Statistics*, Homewood, IL: Richard D. Irwin, Inc., fifth edition.
- Lieberman, G.J. and Resnikoff, G.J. (1955). "Sampling Plans for Inspection By Variables", *Journal of the American Statistical Association*, vol. 50, 457-516.
- United States Department of Defense. (1957). *Sampling Procedures and Tables for Inspection by Variables for Percent Defective* (Mil. Std. 414), Washington, DC: US Government Printing Office.
- United States Department of Defense. (1963). *Sampling Procedures and Tables for Inspection by Attributes* (Mil. Std. 105), Washington, DC: US Government Printing Office.

Appendix

Output 1.

OC Curve Values for MIL-STD-105 Plan
N=2,000, n=125, c=3

OBS	N	C	LOTN	P	K	PA
1	125	3	2000	0.00658	14	0.99109
2	125	3	2000	0.01000	20	0.96775
3	125	3	2000	0.01090	22	0.95577
4	125	3	2000	0.01400	28	0.90698
5	125	3	2000	0.02030	41	0.74849
6	125	3	2000	0.02940	59	0.49010
7	125	3	2000	0.04090	82	0.23296
8	125	3	2000	0.06200	124	0.04055
9	125	3	2000	0.08040	161	0.00659

Output 2.

Sample Size to Achieve Specified PA at AQL
N=2,000, c=1

OBS	LOTN	C	AQL	N	PA	K
1	2000	1	.008	68	0.90187	16

Output 3.

Variable Sample Plan - K Method Procedure

OBS	ALPHA	BETA	P1	P2	K	N
1	0.05	0.1	0.01	0.08	1.80852	26.5908

Output 4.

Variables Sampling Plan - M Method - One Sided Spec Limit

OBS	XBAR	S	L	N	K	PHATL	M
1	18526	754	17000	27	1.809	0.018191	0.031947

Output 5.

OC Curve Values for MIL-STD-414 Plan

OBS	N	K	P	KP	NC	PA
1	30	1.86	0.01	2.32635	12.7419	0.94172
2	30	1.86	0.02	2.05375	11.2488	0.75655
3	30	1.86	0.03	1.88079	10.3015	0.55254
4	30	1.86	0.04	1.75069	9.5889	0.38348
5	30	1.86	0.05	1.64485	9.0092	0.25847
6	30	1.86	0.06	1.55477	8.5158	0.17110
7	30	1.86	0.07	1.47579	8.0832	0.11194
8	30	1.86	0.08	1.40507	7.6959	0.07264
9	30	1.86	0.09	1.34076	7.3436	0.04686
10	30	1.86	0.10	1.28155	7.0193	0.03009
11	30	1.86	0.11	1.22653	6.7180	0.01926
12	30	1.86	0.12	1.17499	6.4357	0.01228
13	30	1.86	0.13	1.12639	6.1695	0.00781
14	30	1.86	0.14	1.08032	5.9172	0.00496
15	30	1.86	0.15	1.03643	5.6768	0.00314

Output 6.

Variable Sampling Plan - M Method - Two Sided Spec Limits

OBS	XBAR	S	L	U	N	K	PHATL	PHATU	M
1	18526	754	17000	20000	27	1.809	0.018191	0.021956	0.031947