

Taguchi Methods in Experimental Design

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Introduction

The quality of a product (or process) is one of the main factors which affects the buying decision of consumers. Dr. Taguchi was one of the pioneers who advocated the use of experimental design for product (or process) quality improvement.

The central idea in Taguchi's philosophy is reduction of variability around a target (or nominal) value. This target is the expected quality characteristic of a product (or process). For example, a 510-gram box of cereal should weigh 510 grams. In this case, the nominal value is 510 grams per box which is how much cereal the consumer expects to get in a 510-gram box of cereal. Taguchi recommends that statistical experimental design methods be employed to reduce variation and move quality characteristics closer to the target. Any departure from this nominal value is a loss to consumer or society.

From Taguchi's point of view, there are three sequential stages for optimizing a product (or process): system design, parameter design, and tolerance design. The system design stage occurs when new concepts or methods are introduced to a new product or process. The parameter design stage is for improving the uniformity of a product. At this stage, Taguchi believes that the performance parameters of a product (or process) should be set to make the performance less sensitive to environmental (uncontrollable) conditions such as road temperature or humidity. Tolerance design stage is for determining the acceptable range of variability around the nominal value determined in the parameter stage. The activities at this stage may include selecting alternative raw materials or operation procedures.

At the parameter design stage, Taguchi utilizes traditional method like ANOVA and orthogonal arrays with a new class of statistics called signal-to-noise ratios in designing an experiment. He calls these types of designs methodology "Taguchi robust design". For implementation of Taguchi Methods, please refer to chapter 6 of SAS/QC software: ADX Menu System for Design of experiments. The following is a brief introduction of Taguchi robust design concept.

Loss Function for Nominal-is-best

Generally, there are three types of loss functions: higher is better, nominal is best, and lower is better. Taguchi imposes the following quadratic loss function for a nominal-is-best situation

$$Loss = k[S^2 + (\bar{y} - n)^2]$$

where k is a constant. $\bar{y}$ and S^2 are the mean and variance of the measurements of quality characteristic respectively, and n is the nominal value (target) of the a product (or process).

The quadratic loss function is made up of two parts: the variance and the squared distance between the mean and the nominal value. In order to minimize the loss to consumer or society, there are three approaches: (1) reduce the variance, (2) move the mean closer to the nominal value, and (3) a combination of both (1) and (2).

Control factors

Control factors are those factors that can be controlled during production and experiment. For example, the types and colors of raw material are control factors in a production process. A design related to the control factors is called the inner array.

Noise factors

Noise factors are those factors that affect product or process performance but are difficult, expensive, or impossible to control. For example, air temperature or vibration may affect automobile carburetor performance, but both of them are impossible to control. A design related to the noise factors is called the outer array.

Example (production of leaf springs):

The goal of the experiment is to investigate four important control factors with one noise factor in a heat treatment process for producing leaf springs of trucks. All factors have two levels (low and high).

Control factor:

TEMP, HEATTIME, TRANTIME, and HOLDTIME

Noise factor:

OILTEMP

Note: For more details, please see page 75 of the text "SAS/QC Software: ADX Menu System for Design of Experiments" and page 154 of the text "Designing for Quality: An Introduction to the Best of Taguchi and ..."

Example (production of elastometric connector):

The goal of the experiment is to find a method to assemble an elastometric connector to a nylon tube that would deliver the required pull-off performance suitable for use in an automotive engine application.

Control factors:

	Levels		
Interference	Low	Medium	High
Connector wall thickness	Thin	Medium	Thick
Insertion depth	Shallow	Medium	Deep
Percent adhesive in connector pre-dip	Low	Medium	High

Noise factor:

	Levels	
Conditioning time	24 h	120 h
Conditioning temperature	72° F	150° F
Conditioning relative humidity	24%	75%

Note: For more details, please see page 535 of the text "Introduction to Statistical Quality Control" by Montgomery.

Signal-to-Noise Ratio

Dr. Taguchi proposed a class of statistics called signal-to-noise ratios (S/N) which can be used to measure the effect of noise factors on the process performance. By maximizing the S/N ratios, the loss functions are minimized. These S/N ratios take into account both the amount of variability and closeness to the average response. In this paper, we will only consider three of them: smaller-is-better, larger-is-better, and target-value-is-best.

Smaller-is-better (variance of response):

This S/N ratio assumes that the target for the response is zero and is appropriate when specifications indicate an upper tolerance limit only.

$$S/N_s = -10 \log\left(\frac{1}{n} \sum y_i^2\right)$$

The goal of an experiment for smaller-is-better situations is to minimize $\sum y_i^2$ and y . That is: we aim to maximize $-10 \log\left(\frac{1}{n} \sum y_i^2\right)$.

Larger-is-better (mean of response):

This S/N ratio assumes that the goal is to maximize the response and is appropriate when specifications indicate a lower tolerance limit only.

$$S/N_L = -10 \log\left(\frac{1}{n} \sum \frac{1}{y_i^2}\right)$$

Again, the goal of an experiment for larger-is-better situations is to maximize the response (e.g., yield of a process). But maximizing y is the same as minimizing $1/y$. This means that we aim to maximize $-10 \log\left(\frac{1}{n} \sum \frac{1}{y_i^2}\right)$.

Target-value-is-best (ratio of mean to variance of response):

This S/N ratio assumes that the given target is best and is appropriate when there is a target value with both upper and lower tolerance limits.

$$S/N_N = 10 \log(\bar{y}^2 / S^2)$$

The goal of an experiment for target-value-is-best situations is to reduce variability around a specific target. When the variability of the response is reduced, relative to the average response, S/N_N will increase.

Critique of Signal-to-Noise Ratio

Many statisticians, like Montgomery and Box (statistical references 4 and 7), point out that by optimizing the S/N ratios, we may not be optimizing the product or process in a physical sense.

Consider the target-value-is-best signal-to-noise ratio:

$$\begin{aligned} S/N_N &= 10 \log(\bar{y}^2 / S^2) \\ &= 10 \log(\bar{y}^2) - 10 \log(S^2) \end{aligned}$$

Hence, maximizing S/N_N for a fixed target value (estimated by $\bar{y}$) is the same as minimizing $\log(S^2)$. By using $\log(S^2)$ directly, we simplify the calculation of S/N_N . This will minimize the chance of computational errors.

Consider smaller-is-better signal-to-noise ratio:

$$\begin{aligned} S/N_S &= -10 \log\left(\frac{1}{n} \sum y_i^2\right) \\ &= -\log\left(\bar{y}^2 + \frac{(n-1)}{n} S^2\right) \end{aligned}$$

The S^2 above is the sample variance. As seen from the above formula, S/N_S directly confounds location and dispersion effects. In addition, this ratio involves y_i^2 or $\bar{y}^2$; hence, it will be sensitive to outliers.

As stated by Montgomery in his text "Introduction to Statistical Quality Control," a better approach for isolating the dispersion (variance) and location (mean) effects is to analyze $\bar{y}$ and $\log(S)$ as separate responses.

Example (mounting integrated circuits):

The goal of this experiment is to maximize the bond strength when mounting an integrated circuit (I.C.) on a metallized glass substrate.

Factors and their levels:

Factor	Low level	High level
A. Adhesive type	D2A	H-1-E
B. Conductor material	copper	nickel
C. Cure time (at 90° C)	90 min.	120 min.
D. I.C. post coating	tin	silver

Experimental design for mounting integrated circuits:

Standard order number	Adhesive type	Conductor material	Cure time	I.C. post coating
1	D2A	copper	90	tin
2	D2A	copper	120	silver
3	D2A	nickel	90	silver
4	D2A	nickel	120	tin
5	H-1-E	copper	90	silver
6	H-1-E	copper	120	tin
7	H-1-E	nickel	90	tin
8	H-1-E	nickel	120	silver

Data for mounting integrated circuits:

Standard order number	Observed response values					mean	deviations	$S/N_L = -10 \log \left(\frac{1}{n} \sum \frac{1}{y_i^2} \right)$
1	73	73.2	72.8	72.2	76.2	73.48	1.5658	37.32
2	87.7	86.4	86.9	87.9	86.4	87.06	0.7092	38.79
3	80.5	81.4	82.6	81.3	82.1	81.58	0.8043	38.23
4	79.8	77.8	81.3	79.8	78.2	79.38	1.4078	37.99
5	85.2	85	80.4	85.2	83.6	83.88	2.0571	38.47
6	78	75.5	83.1	81.2	79.9	79.54	2.9262	38.00
7	78.4	72.8	80.5	78.4	67.9	75.6	5.1676	37.52
8	90.2	87.4	92.9	90	91.1	90.32	1.9942	39.11

From the above table, we see that run number 8 maximizes the S/N_L ratio. Hence, based on the measures of S/N_L only, we should choose the following settings

Adhesive type: H-1-E
Conductor material: nickel
Cure time: 120
I.C. post coating: silver

At these settings, the strength could be maximized and variability could be minimized.

Analyzing Data in the Example (Production of Leaf Springs) by Taguchi Methods

In the Taguchi method, two designs are generated, one for control factors and the other for the noise factors, by selecting suitable orthogonal arrays. The factors in each orthogonal array are orthogonal with each others. Orthogonality means that the effect of one factor does not affect the estimation of the effect of other factors.

In the production of leaf springs example, the control factors are TEMP, HEATTIME, TRANTIME, and HOLDTIME; and the noise factor is OILTEMP. The following Taguchi design is created by repeating the outer array (noise factors) at each design point of the inner array (control factors). The inner array design with 4 two-level factors in eight runs is a fractional factorial resolution IV design (or L_8 orthogonal array). In this case, the signal-to-noise ratio is $10 \log(\bar{y}^2 / S^2)$.

One way to conduct the production of leaf springs experiment is to (1) select one of the eight observations from the inner design array at random; (2) set up the experiment according to the inner array design factor levels; (3) select one of the two levels from the outer array at random and complete the experiment setup; (4) execute the experiment at this combination of inner and outer factor levels and obtain three repeated measurements; (5) select the remaining level from the outer array and complete the experiment setup; (6) execute the experiment at this combination of inner and outer factor levels and obtain three repeated measurements. Finally, repeat all the above steps seven times to obtain all 48 individual observations listed below. Then we can compute the effects of S/N ratio by using SAS/QC software.

The following is a listing of data for the production-of-leaf-springs example:

				OILTEMP	
TEMP	HEAT-TIME	TRAN-TIME	HOLD-TIME	low	high
1840	23	10	2	7.56, 7.62, 7.44	7.18, 7.18, 7.25
1840	23	12	3	7.50, 7.56, 7.50	7.50, 7.56, 7.50
1840	25	10	3	7.94, 8.00, 7.88	7.32, 7.44, 7.44
1840	25	12	2	7.78, 7.78, 7.81	7.50, 7.25, 7.12
1880	23	10	3	7.56, 7.81, 7.96	7.81, 7.50, 7.59
1880	23	12	2	7.59, 7.56, 7.75	7.63, 7.75, 7.56
1880	25	10	2	7.69, 8.09, 8.06	7.56, 7.69, 7.62
1880	25	12	3	8.15, 8.18, 7.88	7.88, 7.88, 7.44

Basically, there are three steps in the Taguchi analysis: (1) find the factors that affect the S/N ratio and set the levels of these factors to satisfy our objective, (2) find the factors that affect the average response and set the levels of these factors toward to the target value, and (3) find the factors and set their levels to best satisfy both 1 and 2 above and other practical issues. In other words, we actually analyze the data twice: first we analyze the effect of standard S/N ratio, next the effect of average response.

Step by step instructions (from entering data to producing effect estimates) are clearly stated in chapter 6 of the text "SAS/QC Software: ADX Menu System for Design of Experiments". In this paper, we will only go over the interpretation of the outputs.

Output for the SAS/QC ADX Menu System

ADX: Effect Estimates (from analyzing the S/N Ratio)

Effect	Estimate	P-value
TEMP	-0.16724	0.9360
HEATTIME	-4.63413	0.0945
TRANTIME	2.284202	0.3193
HOLDTIME	1.470349	0.4991

From the above output, we see that the P-value associated with HEATTIME is relatively small, this means HEATTIME has an effect on the S/N ratio. Since its effect estimate is negative, we should choose the low level of HEATTIME (i.e., 23 seconds) to maximize the S/N ratio.

ADX: Effect Estimates (from analyzing the average response)

Effect	Estimate	P-value
TEMP	0.110625	0.0032
HEATTIME	0.088125	0.0061
TRANTIME	0.014375	0.3394
HOLDTIME	0.051875	0.0264

From the above output, we see that the P-values associated with TEMP, HEATTIME, and HOLDTIME are all quite small. This means that they all have a strong effect on the average leaf spring height. Since their effect estimates are all positive, we should choose the high level of the factors to maximize the average unloaded height if our objective is maximization.

Discussion

At this point, we have seen some main components of the Taguchi methods in the design of an experiment. The pros and cons of the Taguchi methods are the following:

1. The three sequential stages for optimizing a product or process have won the attention of a new audience.
2. Implementing the robust design in the parameter design stage produces results.
3. Formulating the loss associated with variability by the loss function helps the researchers see that there is a cost attached to variance .
4. The concept of simultaneous study of both the mean and variance has established new directions for quality-engineering research.
5. Many of the inner array designs and outer array designs are simple two-level fractional factorials design that may have messy alias structures.
6. The inner array and outer array structures may lead to a very large experiment.
7. Some of the S/N ratios (e.g., the S/N_s) confound the location and dispersion effects.
8. Assumptions underlying the analysis part of robust design are not stated.

Conclusion

Dr. Taguchi made experimenters aware of the significance in using experimental designs to locate factors and their levels that affect both the average and the variance of the process response. Before Dr. Taguchi introduced his idea, most experimenters used experimental designs (e.g., fractional factorial designs) to assess the effect on averages only. The idea of robust experiment is sound; however, we need to differentiate the two components (i.e., first, the experimental design and second, the analysis) of the robust design procedure. We may find other analysis techniques, rather than a strict S/N ratios transformation, in the analysis part to account for the effect of variance. By carefully implementing the robust design, new products or processes with enhanced field performance and reliability can be discovered. Also, companies can reduce costs and improve quality at every stage of product development.

SAS References:

1. SAS/ASSIST Software Your Interface to the SAS System (1990), Version 6, Cary, NC SAS Institute.
2. Getting Started with the SAS System Using SAS/ASSIST Software (1990), Version 6, Cary, NC SAS Institute.
3. Doing More with SAS/ASSIST Software (1990), Version 6, Cary, NC SAS Institute.
4. SAS/QC Software: ADX Menu System for Design of Experiments (1990), Version 6, Cary, NC SAS Institute.

Statistical References:

1. Box, G.E.P; Hunter, W.G.; and Hunter, J.S. (1978), Statistics for Experimenters, New York: John Wiley & Sons, Inc.
2. Lochner, R.H. and Matar, J.E. (1990), Designing for Quality: An Introduction to the Best of Taguchi and Western Methods of Statistical Experimental Design, White Plains: Quality Resources.
3. Montgomery, D.C. (1991), Design and Analysis of Experiments, 3d ed., New York: John Wiley & Sons, Inc.
4. Montgomery, D.C. (1985), Introduction to Statistical Quality Control, 2d ed., New York: John Wiley & Sons, Inc.
5. Wadsworth, H.M., Stephen, K.S., and Godfrey, A.B. (1986), Modern Methods for Quality Control and Improvement, New York: John Wiley & Sons, Inc.
6. Peace, Glen Stuart (1993), Taguchi Methods: a Hands-on Approach, Addison-Wesley Publishing Co., Inc.
7. E. P. Box (1985), Discussion, Off-line Quality Control, Parameter Design, and the Taguchi Method, Journal of Quality Technology, vol. 17, no. 4, pp. 189-190, 1985.