

Analysis of Growth Curves

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ABSTRACT

Researchers in psychology often use longitudinal designs to assess change. Various statistical techniques have been used to analyze these data, including repeated measures analysis of variance, multivariate analysis of variance, and multiple regression. This paper illustrates the use of the SAS system to examine growth curves with these methods and introduces a relatively unused technique, linear mixed models, as well. The advantages of using a mixed model approach to growth curve analysis are discussed.

INTRODUCTION

Longitudinal studies are common in psychological research, especially in developmental psychology. A variety of methods are employed to analyze change across time, including repeated measures analysis of variance, multivariate analysis of variance and regression (Collins & Horn, 1991; Nesselroade & Baltes, 1979). These methods allow researchers to assess whether change occurs across time, as well as how the change occurs. However, real data often do not conform to the assumptions and restrictions required by these methods. A relatively unused method, linear mixed models, offers researchers a procedure to examine growth while avoiding many of the drawbacks that many methods present.

In this paper, we illustrate the use of SAS to analyze growth curves. Using real data, repeated measures ANOVA, MANOVA, multiple regression, and mixed models analyses will be performed.

DATA

The data we use are from a study on the acquisition of hand preference in infants at ages seven, nine, eleven and thirteen months. Information regarding the hand preference of both the mother and father was also collected. The handedness of the mother is used as a between-subjects variable. So, this is a one-between (three levels), one-within design, with four repeated observations. We are interested in examining the change in hand preference across the four observations.

The first ten subjects' measurements are provided:

ID	TIME1	TIME2	TIME3	TIME4	MOMHAND
1	3.67	1.04	-2.06	-0.96	1
2	2.29	1.88	1.18	2.89	3
3	-1.28	-1.96	-3.78	2.65	3
4	-0.89	-1.46	-1.96	-2.4	2
5	2.84	0.23	2.83	1.21	3
6	0.41	-2.13	1.83	-0.22	3
7	-1.6	0.47	1.13	3.53	3
10	4.02	3.16	2.12	3.65	3
11	1.15	2.13	1.7	1.15	3
12	1.04	0.65	-2.29	2.12	3

ANALYSIS OF GROWTH CURVES

These data can be examined through univariate and multivariate procedures. We will illustrate the use of repeated measures analysis of variance, multivariate analysis of variance, multiple regression, and mixed models in SAS.

REPEATED MEASURES ANALYSIS OF VARIANCE

Repeated measures analysis of variance (ANOVA) is useful in that it can detect variation in measurement across multiple observations, as well as identify differences between groups on the characteristic of interest. As development of cognitive function is often studied as a change in skill across time, and these data are often analyzed using univariate repeated measures ANOVA. In our example, change in the within-subjects variable, handedness score, can be examined using maternal handedness (left preference, no preference, and right preference) as the between-subjects factor.

In SAS, repeated measures ANOVA is performed using PROC GLM. The SAS code follows:

```
proc glm data = hand ;  
  class momhand ;  
  model time1 time2 time3 time4 = momhand  
    / nouni ;  
  repeated time 4 (7 9 11 13) / printe ;  
run ;
```

The PROC GLM statement invokes the procedure, using the data "hand." The CLASS statement identifies the variable momhand as the categorical between-subjects factor. The within-subjects variable, the handedness scores, are examined by the grouping variable in the MODEL statement. NOUNI suppresses univariate tests for differences at each time point. As we are interested in overall change, we use this option. Finally, the REPEATED statement identifies the within-subjects factor and its levels and the PRINTE option displays the results for the test of compound symmetry.

The code produces these results (in all sections, the output has been edited to save space):

```
Test for Sphericity:  
Mauchly's Criterion = 0.2988  
Chisquare Approx = 54.023587 with 5 df  
Prob > Chisquare = 0.0000
```

```
Applied to Orthogonal Components:  
Test for Sphericity:  
Mauchly's Criterion = 0.8696731  
Chisquare Approx = 6.2449185 with 5 df  
Prob > Chisquare = 0.2831
```

```
General Linear Models Procedure  
Repeated Measures Analysis of Variance  
Tests of Hypotheses for Between Subjects  
Effects
```

Source	SS	Mn Sq	F	p
MOMHAND	54.435	27.217	1.99	0.1489
Error	630.672	13.710		

Univariate Tests of Hypotheses for Within Subject Effects

Source: TIME

SS	Mn Sq	F	p	Adj G - G	p H - F
10.965	3.655	0.88	0.45	0.4464	0.4546

Source: TIME*MOMHAND

SS	Mn Sq	F	p	Adj G - G	p H - F
48.117	8.019	1.92	0.080	0.0886	0.0809

Source: Error(TIME)

SS	Mn Sq
574.93	4.166
Greenhouse-Geisser Epsilon = 0.9085	
Huynh-Feldt Epsilon = 1.0133	

These results indicate that, overall time does not have an effect on the acquisition of handedness preference ($F = 0.88$, $p = 0.45$). There is not an overall effect for mother's hand preference, either ($F = 1.99$, $p = 0.1489$). The interaction between time and mother's handedness approaches significance ($F = 1.92$, $p = 0.08$).

These results should be interpreted with caution, however. First, the data are very unbalanced. The numbers of subjects in the categories of maternal handedness are very unequal. Forty-one infant have mothers who are right-handed, while only five infants have left-handed mothers. In controlled experiments, the researcher has more control over the between-subjects variable. They can dictate the number of subjects in each condition. These data illustrate a problem that many longitudinal researchers encounter.

Also, the assumption of compound symmetry has been violated in these data (Mauchly's Criterion = 0.2988, $\chi^2 = 54.02$, $p < 0.0005$). Repeated measures ANOVA assumes that the covariance matrix has a compound symmetry structure; that is, the correlations between all measurements are equal. This assumption often does hold for controlled experiments, but it is often violated in longitudinal data. Since the measurements are taken in a fixed order in the same person, consecutive measurements are likely to be more highly correlated than measurements further apart in time. The violation of the restrictive compound symmetry assumption suggests that a method with weaker assumptions may be a better option.

MULTIVARIATE ANALYSIS OF VARIANCE

Unlike the repeated measures ANOVA, MANOVA puts no restrictions on the structure of the covariance matrix, making this method an attractive option for growth curve analysis (Guire & Kowalski, 1979). Again, we use PROC GLM for the analysis:

```
proc glm data = hand ;
  class momhand ;
  model time1 time2 time3 time4 =
    momhand / nouni ;
  repeated time 4 / summary ;
run ;
```

The results:

Manova Test Criteria and Exact F Statistics for the Hypothesis of no TIME Effect
H = Type III SS&CP Matrix for TIME E = Error SS&CP Matrix

Statistic	Value	F	p
Wilks'			
Lambda	0.9286	1.1281	0.3481
Pillai's			
Trace	0.0714	1.1281	0.3481
Hotelling-			
Lawley Trace	0.0769	1.1281	0.3481
Roy's			
Greatest Root	0.0769	1.1281	0.3481

Manova Test Criteria and F Approximations for the Hypothesis of no TIME*MOMHAND Effect
H = Type III SS&CP Matrix for TIME*MOMHAND
E = Error SS&CP Matrix

Statistic	Value	F	p
Wilks'			
Lambda	0.7597	2.1594	0.0545
Pillai's			
Trace	0.2479	2.1224	0.0583
Hotelling-			
Lawley Trace	0.3060	2.1929	0.0513
Roy's Greatest			
Root	0.2682	4.0225	0.0128

The results of the MANOVA indicate that time by itself does not affect the acquisition of hand preference (Wilk's Lambda = 0.9286; $F = 1.2181$, $p = 0.3481$). The interaction between maternal hand preference and time approaches significance (Wilk's Lambda = 0.7597; $F = 2.1594$, $p = 0.0545$). Having a mother with a particular hand preference influences an infant's acquisition of hand preference over time.

MULTIPLE REGRESSION

If a researcher is interested more in how change occurs across time, multiple regression can be used. Performing a separate regression on each of the groups provides a picture of how each group develops over time. The researcher can determine whether a linear, quadratic, or cubic curve best describes the development of the characteristic.

For each set of infants, those with mothers who are left-handed, right-handed, or neither, a regression was run to examine the pattern of scores. In looking at plots of the groups, it seems that a linear or cubic relationship best describe the data. So, a variable representing a cubic relationship to time needs to be generated:

```
data hand;
  set hand;
  time3 = time*time*time;
```

Also, the data needs to be restructured so that each score has its own line. Therefore, each subject will show multiple lines of data:

ID	SCORE	TIME	TIME2	TIME3
1	3.67	7	49	343
1	1.04	9	81	729
1	-2.06	11	121	1331
1	-0.96	13	169	2197

2	2.29	7	49	343
2	1.88	9	81	729
2	1.18	11	121	1331
2	2.89	13	169	2197
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
12	1.04	7	49	343
12	0.65	9	81	729
12	-2.29	11	121	1331
12	2.12	13	169	2197

Then, for each group, the score is then regressed on the linear and cubic time variables:

```
proc reg data = hand;
  model score = time time3;
run;
```

For infants with left-handed mothers, the results are:

Analysis of Variance

Source	Sum of Squares	Mean Square	F	p
Model	36.91876	18.4593	3.328	0.0603
Error	94.30673	5.54745		
C Total	131.22550			

Root MSE	2.35530	R-square	0.2813
Dep Mean	0.14450	Adj R-sq	0.1968
C.V.	1629.96786		

Parameter Estimates

	Param Est	Std Error	T	p
INTRCP	21.885	8.808	2.485	0.0237
TIME	-3.2758	1.376	-2.388	0.0288
TIME3	0.0096	0.004	2.185	0.0432

For the no-preference group:

Analysis of Variance

Source	Sum of Squares	Mean Square	F	p
Model	0.09794	0.0489	0.011	0.9893
Error	40.9411	4.5490		
C Total	41.03910			

Root MSE	2.13284	R-square	0.0024
Dep Mean	-0.56500	Adj R-sq	-0.2193
C.V.	-377.49428		

Parameter Estimates

	Param Est	Std Error	T	p
INTRCP	-1.0529	10.2969	-0.102	0.9208
TIME	0.0538	1.60355	0.034	0.9740
TIME3	-0.00004	0.00512	-0.009	0.9934

For the right-handed group:

Analysis of Variance

Source	Sum of Squares	Mean Square	F	p
Model	2.79883	1.39941	0.301	0.7405
Error	748.653	4.65002		

C Total 751.45228

Root MSE	2.15639	R-square	0.0037
Dep Mean	0.75305	Adj R-sq	-0.0087
C.V.	286.35473		

Parameter Estimates

	Param Est	Std Error	T	p
INTRCP	-1.3468	2.8160	-0.478	0.6331
TIME	0.3169	0.4386	0.723	0.4709
TIME3	-0.0009	0.0014	-0.664	0.5079

Only for the group with left handed mothers does the model approach significance ($F = 3.328$, $p = 0.0603$). There seems to be both a linear and cubic component to the change across time.

A consistent linear or cubic pattern of change is not discernible for the other two groups. Neither of the models tested are significant. Analysis of the growth curves at the individual level may be more informative.

INDIVIDUAL REGRESSIONS

Rather than concentrating on group averages, examining growth curves at the individual level allows a more detailed description for the subjects than other approaches. Individual differences are emphasized over group differences.

The SAS code to run individual regressions is:

```
proc reg data=hand tableout outest=indiv
  noprint ;
  by id ;
  model score = time time2 ;
run ;
```

PROC REG calls the regression procedure on our data. The TABLEOUT option in the PROC REG statement will cause the standard errors, t-values and associated probabilities to be included in the OUTEST = indiv data set. We have suppressed the individual ANOVA and parameter estimates tables with the NOPRINT option. Because we are interested in describing change, the significance of the model and the estimates is not as important. For this reason, we also display only the parameter estimates themselves, and not the t-values, standard errors and p-values. The code to accomplish this is:

```
proc sort data = indiv ;
  by _type_ ;
run ;

data indiv ;
  set indiv ;
  keep id intercep time time2 ;
  if _N_ > 51 then delete ;
run ;

proc print data = indiv ;
run ;
```

We use PROC SORT to arrange the data according to the output type (parameter, p-value, standard error, or t-value). Since we only want the parameter estimates, we delete all other information in the data. Finally, the beta estimates for each individual are displayed.

ID	INTERCEPT	TIME	TIME2
1	3.9035	-4.4965	0.9325
2	2.4250	-1.4800	0.5300
3	-0.8105	-4.3355	1.7775
4	-0.8905	-0.6005	0.0325
5	2.3685	-0.9715	0.2475
6	-0.2155	-0.1605	0.1225
7	-1.4425	1.3575	0.0825
10	4.1575	-2.0075	0.5975
11	1.2145	1.1045	-0.3825
12	1.5350	-3.5700	1.2000

The estimates for the first 10 infants are shown.

All of the preceding methods for analyzed growth curves are frequently used, and offer researchers a variety of options depending on the focus of the research question. However, they do have their drawbacks. Repeated measures ANOVA assumes a covariance structure that is not likely to be met by longitudinal data. While MANOVA avoids this problem, the covariance structure is not likely to be unstructured, either. Ideally, the researcher could specify the covariance structure in the analysis.

Also, ANOVA, MANOVA, and regression all require the data are balanced. All subjects must provide complete cycles of measurements, or the entire case must be excluded. This is a problem for longitudinal researchers as subjects often miss observations, or are not available at precisely the time that the researcher would like. So, either because of missing data or variation in time of measurement, data are often unbalanced.

LINEAR MIXED MODELS

The use of linear mixed models has been illustrated in educational psychology (Singer, 1998), but has been rare in other areas of psychology to date. Previously, analyses using mixed models could only be performed with specialized software such as MLn (Prosser, Rabash, & Goldstein, 1996) or HLM (Bryk, Raudenbusch, & Congdon, 1996). Consequently, most psychologists did not have access to this methodology. With the inclusion of PROC MIXED in SAS, it is more feasible for psychologists to use than it was previously.

To avoid the problems that other methods present in growth curve analysis, researchers should using linear mixed models. Using PROC MIXED, a researcher can analyze growth curve data without excluding cases, and specify the covariance structure.

Modeling growth curves with PROC MIXED extends the individual regression analysis, allowing one to model more than one error term. This approach analyzes the data at more than one level. the repeated observations on an individual comprise the first (within-subject), and the subjects themselves comprise the second (between-subjects). In our data, the repeated assessments of hand preference are level-one variables. Maternal hand preference is a person-level factor, making it a level-two variable. Regression coefficients computed for individual subjects are considered the outcome variables in a subsequent regression (Kreft, de Leeuw, & van der Leeden, 1994). This model formulation contains fixed (time and maternal hand preference) and random (intercept, time and individual variation) parts.

As for the regression example, the data must be in a specific format for PROC MIXED to use it. Each subject needs to have multiple lines of data, and one response variable:

It is important to note that to fit growth curves with SAS PROC MIXED, the data cannot be in this format. Rather, each subject will have multiple lines of data representing each observation. The resulting data should look like this:

ID	SCORE	MOMHAND	TIME
1	3.67	1	0
1	1.04	1	1
1	-2.06	1	2
1	-0.96	1	3
.	.	.	.
.	.	.	.
.	.	.	.
12	1.04	3	0
12	0.65	3	1
12	-2.29	3	2
12	2.12	3	3

It is this structure which allows the inclusion of subjects who did not provide full cycles of measurement. The line of data for which a measurement is not available is deleted. For these data, we can analyze 153 subjects, whereas we were only able to analyze 46 subjects with the other methods.

First we explore the covariance structure of the data, specifying the matrix first as unstructured. We use the following code:

```
proc mixed method = REML data = hand ;
  class id momhand;
  model score = time momhand/ noint
    solution
    predicted;
  random intercept time / type = un
    subject = id solution;
  repeated time / subject = id group =
    momhand;
run ;
```

We use PROC MIXED to call on the mixed models procedure, specifying the restricted maximum likelihood method for estimation. Because we are modeling individual growth curves, we name id and mother's handedness as categorical variables in the CLASS statement. Fixed effects are specified in the MODEL statement. In this example, we examine score as a function of time and maternal handedness. The SOLUTION option will display the parameter estimates for the fixed effects. Also, the predicted values for each individual's scores will be displayed, as requested by the PREDICTED option.

Random effects are defined in the RANDOM statement. Here, we allow the intercept and time effects to be random by including them in this statement. The RANDOM statement is also where one can specify the covariance structure. Here we have allowed the matrix to remain unstructured (TYPE = un). We also request the random effects estimates with SOLUTION.

The REPEATED statement defines the order of measurements within each subject. The TIME variable is used to order each handedness score within the subject.

The results are:

The MIXED Procedure

Covariance Parameter Estimates (REML)

Cov Parm	Subject Group	Estimate
UN(1,1)	ID	1.70550473
UN(2,1)	ID	-0.27618673
UN(2,2)	ID	0.23602477
DIAG	ID Mom: Left	5.38585868
DIAG	ID Mom: No Pref	3.10133543
DIAG	ID Mom: Right	3.77260449

Model Fitting Information for SCORE

Description	Value
Observations	494.0000
Res Log Likelihood	-1106.28
Akaike's Information Criterion	-1112.28
Schwarz's Bayesian Criterion	-1124.86
-2 Res Log Likelihood	2212.558
Null Model LRT Chi-Square	25.2874
Null Model LRT DF	5.0000
Null Model LRT P-Value	0.0001

Solution for Fixed Effects

Effect	Estimate	Std Error	t	p
TIME	0.020965	0.1020029	0.21	0.8374
Left	0.617832	0.4390933	1.41	0.1610
No pref	0.552037	0.4645946	1.19	0.2362
Right	0.881904	0.1876712	4.70	0.0001

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TIME	1	150	0.04	0.8374
MOMHAND	3	192	7.55	0.0001

These results indicate that time is not a factor in the change of hand preference ($F = 0.04$, $p = 0.8437$). Since time is not influential, we will next try a simple covariance structure, where the covariances are all set to zero. The fit statistics of the two models will be compared.

The SAS code is very similar, with only the change to the TYPE = option in the RANDOM statement:

```
proc mixed method = REML data = hand ;
  class id momhand ;
  model score = time momhand / noint
    solution predicted ;
  random intercept time / type = simple
    subject = id solution ;
  repeated / subject = id group = momhand;
run ;
```

The results:

Covariance Parameter Estimates (REML)

Cov Parm	Subject Group	Estimate
INTERCEPT	ID	1.23506558
TIME	ID	0.07587702
DIAG	ID Mom: Left	5.69343392
DIAG	ID Mom: No pref	3.11821503
DIAG	ID Mom: Right	3.97610155

Model Fitting Information for SCORE

Description	Value
Observations	494.0000

Res Log Likelihood	-1106.76
Akaike's Information Criterion	-1111.76
Schwarz's Bayesian Criterion	-1122.25
-2 Res Log Likelihood	2213.526

Solution for Fixed Effects

Effect	Estimate	Std Error	t	p
TIME	0.020388	0.09791930	0.21	0.8353
Left	0.621569	0.43950782	1.41	0.1589
No Pref	0.591655	0.45653233	1.30	0.1965
Right	0.879190	0.18155312	4.84	0.0001

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TIME	1	150	0.04	0.8353
MOMHAND	3	192	8.11	0.0001

The two models can be compared. For the fit values a larger value indicates a better model. We will compare the -2 Log Likelihood (-2LL), Akaike's Information Criterion (AIC), and Schwarz's Bayesian Criterion (SBC).

	Unstructured	Simple
-2LL	2212.558	2213.526
AIC	-1112.28	-1111.76
SBC	-1124.86	-1122.25

All three statistics increased slightly with the simple covariance structure. We will use this structure in evaluating the parameters estimates in the next model.

The SAS code for the final model is similar to the previous model. However, in looking at parameter estimates, we will switch from restricted maximum likelihood estimation to maximum likelihood estimation (Wolfinger, 1993).

```
proc mixed method = ML data = hand ;
  class id momhand ;
  model score = time momhand / noint
    solution predicted ;
  random intercept time / type = simple
    subject = id solution ;
  repeated / subject = id group = momhand;
run ;
```

The results follow:

Covariance Parameter Estimates (MLE)

Cov Parm	Subject Group	Estimate
INTERCEPT	ID	1.19127394
TIME	ID	0.06802556
DIAG	ID Mom: Left	5.62620631
DIAG	ID Mom: No Pref	3.09129492
DIAG	ID Mom: Right	3.98507532

Model Fitting Information for SCORE

Description	Value
Observations	494.0000
Log Likelihood	-1104.50
Akaike's Information Criterion	-1109.50
Schwarz's Bayesian Criterion	-1120.01
-2 Log Likelihood	2209.002

Solution for Fixed Effects

Effect	Estimate	Std Error	t	p
TIME	0.019036	0.0975368	0.20	0.8455
Left	0.621127	0.4344735	1.43	0.1545
No Pref	0.590526	0.4508445	1.31	0.1918
Right	0.881030	0.1805665	4.88	0.0001

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TIME	1	150	0.04	0.8455
MOMHAND	3	192	8.22	0.0001

The results of the analysis indicate that mother's handedness ($F = 8.22$, $p = 0.0001$) is an influential factor infant handedness while time is not ($F = 0.04$, $p = 0.8455$). Specifically, an infant with right-handed mother is most influenced ($\beta = 0.881$; $t = 4.88$, $p = 0.0001$).

However, the covariance estimates that there is much variability in the random effects. The intercept's variance is 1.19 and the slope's (time) is 0.068. The largest source of variation is the individual. These estimate range from 3.09 to 5.62.

CONCLUSION

Psychologists often employ longitudinal designs in their research. Traditionally, univariate and multivariate ANOVA models and multiple regression have been used to examine change across time. We used a real world data to illustrate the analysis of growth curves through these procedures.

We also illustrated linear mixed models by SAS PROC MIXED as an alternative that psychologists should consider. Linear mixed models provide researchers with increased flexibility in modeling growth curves, and allows the inclusion of unbalanced data in the analysis. As these present major problems in longitudinal research, a mixed models approach is an attractive one for researchers.

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