

Distance Between Two Geographic Locations

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One often needs to obtain the distance between two locations (often between two zipcodes), given only their geographic coordinates (latitude and longitude). Some programmers try to solve this by using a little high-school geometry, some sort of variation on $\text{distance} = \sqrt{x^2 + y^2}$. The reason why this doesn't work except possibly over quite short distances is that this Euclidean solution is only correct on a plane. However, the points are on the Earth's surface, which is nearly spherical.

A solution follows which would be exactly correct if the Earth were a perfect sphere. The inaccuracy resulting from the departure from sphericity is quite small, and the following expression works nearly as well as more complicated ones. (I detest MS Word's equation editor and the conference organizers couldn't accept LaTeX, so I'll render this in SAS notation and let the lines wrap where they may.)

```
miles=3953*arccos(sin(phi1)*sin(phi2)
)+cos(phi1)*cos(phi2)*cos(long1-
long2));
```

In this equation, "phi1" and "phi2" are the latitudes of the place1 and place2, and "long1" and "long2" are the respective longitudes. SAS's trigonometric functions (as is the case with most other languages) require that the phi's and long's be converted to radians; divide degrees by 57.29578 to get radians. Make sure not to confuse degrees and minutes with decimal degrees.

North latitudes and west longitudes are positive, and south latitudes and east longitudes are negative. (The signs of east and west can be reversed if convenient,

as long as longitudes in the same hemisphere always get the same sign.) Persons adept at trigonometry will note that it really doesn't matter which place is "place 1" and which is "place 2".

Distances, no matter how computed, are of course in airline miles; the distance by road is always longer, usually by a lot.

Here's a sample program.

```
data _null_;
  input place1 $ phi1 long1 place2
    $ phi2 long2;
  retain radians 57.29578;
  phi1=phi1/radians;
  phi2=phi2/radians;
  long1=long1/radians;
  long2=long2/radians;
  miles=round(3953*arccos(sin(phi1)
    *sin(phi2)+cos(phi1)*cos(phi2)
    *cos(long1-long2)));
  put place1 'is ' miles
    'miles by air from ' place2;
  cards;
  Seattle 47.6 122.3 NYC 40.5 74
  Portland 45.5 122.7 Paris 48.8 -2.3
run;
```

The output from the log is:

```
Seattle is 2404 miles by air from
NYC
Portland is 5121 miles by air from
Paris
```

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