

A System to Calculate Market Value-at-Risk using Oracle® and Monte Carlo Simulation in SAS/IML®

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ABSTRACT

SAS/IML® is used with Base SAS and Oracle® to produce a system to calculate Value-at-Risk (VaR) with the flexibility to reflect changes in the database, in the calculation, and reporting. VaR is calculated using Monte Carlo simulation. The overall process is covered and aspects of the calculation are highlighted. Finally, the impact of the results on the Enterprise are outlined.

INTRODUCTION

Three years ago upper management at GMAC Mortgage Group decided that it was important to establish policies and programs designed to monitor the market, credit, and operational risks the three companies of the Group faced. The program described here was written to measure the risks of market assets of the enterprise. SAS/IML® greatly facilitated the task and the elegance of the resultant code has given transparency to the process.

WHAT IS VALUE-AT-RISK?

Value-at-Risk is the maximum amount of money that may be lost on a portfolio on average over a given period of time, with a given confidence interval *under normal circumstances*. For traders this period of time, or holding period, can be a day or less. For our model the holding period is one month. When we say that our VaR is -\$47m for a given company at the 95th confidence interval, we mean that, under normal markets conditions, we expect to lose more than \$47m, 5% of the time. Note that this does not consider catastrophic events.

VALUE-AT-RISK at GMAC

While there are various ways of calculating Value-at-Risk, we use a two factor, interest rate and spread, correlation model. To help explain the mechanics of the model, I've illustrated a simple example involving three risk factors, three products, running 10 simulations in Appendix A.

PRODUCT PROFILES

The model assumes that each product is at risk because of movements in interest rates and spreads, and we associate an IR security to each product that makes sense, such as the 5 year Treasury bond, and use that as the products interest rate risk factor. We also associate another series as the spread factor, which is the difference in bid and ask price of that product. We measure the historical movement of all such risk factors by calculating the standard deviation over a 60 month period. As we see on the right side of the chart, these standard deviations or volatilities are used by traders at their desks in models to "shock" the corresponding products to produce product profiles. Please note that I've subtracted the base value (the nominal value) from all seven elements of the profiles, as we're interested in the amount we expect to gain or lose. In our case each product has two profiles, one for interest rate, one for spread, that reflect the dollar value of the position in response to up and down three standard deviation, or sigma, moves in the associated risk factor. From these profiles we see how a given movement in interest rate can cause a gain in one product and a loss in another.

Traders are accustomed to conducting such sensitivity analysis and we continued to use their input, even though this was less efficient than performing such analysis in the model. We also worried that the process could produce inconsistent profiles; two traders at different companies could use different programs to shock their products or use different assumptions in the same model and derive different profiles for the same product. In fact, unexpected VaR results prompted market managers to review assumptions in these models. Allowing traders retain this function helped us win confidence in our results.

RISK FACTOR SIMULATION

On the left side of the chart, we note that the 60 period risk factor table is used to calculate a correlation and covariance table. From the correlation table we see the extent to which each risk factor move with or against each other risk factor. The Singular Value Decomposition function in IML uses the covariance matrix of risk factors, the number of simulations and the random number generator to create a matrix of simulated risk factors. It's a simple matter to verify that the SVD function is working: computing the covariance matrix of the simulated risk factor matrix produces the same covariance matrix as that calculated from the historical series.

Each element of the simulated matrix is standardized by the corresponding risk factor's standard deviation. The standardized risk factor matrix consists of units (sigma's) that can be compared to the product profiles the traders have provided. We create profit and loss matrices, one for interest rate, one for spread, by interpolating each product's risk factor 's column of simulated sigma's (standardized) against that product's profile. This results in two columns (one for interest rate, one for spread) of profit and loss values for each product; the number of rows corresponds to the number of simulations.

We can see that these profit and loss results are correlated because they are determined using risk factors that are correlated. Add the interest rate and spread profit and loss tables to obtain the total profit and loss results. Add all of these columns (products) laterally to get a single column of profit and loss; sort this column in descending order, VaR at the 95% confidence interval is the 95th percentile (or 5th percentile from the bottom) of that column.

VaR can be calculated similarly for any level of aggregation, or even at the product level. Simply add all the columns that comprise a given company, or business unit, etc., across and determine the 95th (or 99.9th or 99.95th) percentile of the resulting column. The main point here is that, because the elements within each row of the profit and loss matrix are correlated, one cannot sum product level VaRs and obtain the total VaR. Theoretically, it is possible that a level of aggregation can equal the sum of the products, but typically the diversification benefit, the difference in the sum of the product VaR's to the total VaR, caused by the difference in profiles of the products and the correlation of the risk factors, is quite significant. One must choose those products to be included with the level of aggregation desired and sum the corresponding columns into a single column, which is then used to determine VaR at the various confidence intervals.

THE OLD MODEL

Three separate installations of a system to calculate VaR existed at the three companies that comprise the GMAC Mortgage Group. These systems had been written in VBA in MS Access. The system had serious drawbacks:

- Without extensive rewrites to the code the system could not accept new inputs in either risk factors nor products
- the system was unable to replicate runs for previous periods without removing current data from the database
- it was slow
- it didn't produce a total VaR number for the entire Mortgage Group
- the methodology was buried in confusing code

The task was to rewrite this existing system to correct these shortcomings. A database in Oracle was designed to accommodate all elements necessary for a Mortgage Group level VaR calculation. SAS/Base and SAS/IML were chosen to create an efficient, flexible and transparent system.

ORACLE DATABASE

The database is designed to:

- maintain description of the variables used in the model
- accommodate expansion in the reporting structure
- validate the data

The database schema is illustrated in Appendix B. The reporting structure reflects the organization of the Enterprise, company group, company, business, business unit, and desk. The desk summarizes the structure and each product is assigned to a single desk, i.e., product numbers are assigned such that single product cannot appear in two different desks.

While there are several tables to maintain audit trails and aid in special calculations, we need to focus on just a few tables for our discussion. The monthly risk factor updates are stored in the risk_factor_value table and are described in the risk_factor table. As this database is also the repository for the counterparty system, this descriptive table is used to select only risk factors the VaR model uses. The monthly profiles and associated risk factors are entered into the product_market_value table and these products are described in the product table. Referential integrity ensures that a risk factor exists for each risk factor entered into the product_market_value table. Finally we maintain the results for all business unit levels, for all confidence intervals, for all months reported in the var_result table.

THE NEW MODEL

As the existing model based much of its assumptions on J.P Morgan's *Risk Metrics - Technical Document* that required coding customized statistics, SAS/IML was chosen for the rewrite.

VOLATILITIES

We assume that the distribution of the interest rate risk factors to be log normal and that of spread to be normal; we use the log of the ratio of adjacent values for interest rate factor and the difference of adjacent values for spread. For computational ease interest rate and spread factors were read from the database and denormalized into separate datasets using proc transpose. The risk_factor_id's , a numeric field that becomes the column names after proc transpose, e.g., values for risk_factor_id 12 appear under the column header of "_12." We will need to retain this series to use to label our results (we repeat the following code for spread factors as well):

```
proc transpose data = risk_val_IR
  out = risk_val_IR_flat
  (drop=_NAME_ _LABEL_ valdat) ;
id risk_factor_id ;
by descending valdat ;

proc sql ;
%global risk_number_IR_list ;
select input(substr(name,2,3),3.)
  into :risk_number_IR_num separated by ','
  from dictionary.columns
  where libname='WORK'
  and memname='RISK_VAL_IR_FLAT' ;
quit ;
```

The methodology calls for a dampening effect of .97 for monthly data to be incorporated into the calculation of the standard deviation, i.e., a weighting scheme such that more recent values count for values further in the past. This was accomplished with very few lines of code compared to the original model (maxrow is the number of periods back we want to go, which we initially had to vary for lack of data). First, we transform our interest rate risk factors and calculate the vector of weights:

```
IR_trans = log(dataAll_IR[1:&maxrow-1,]) - log(dataAll_IR[2:&maxrow,]) ;
back = &maxrow - 1 ;
lambda = 0.97 ;
index = (1:back)` ;
w = (1-lambda) * lambda**(index-1) / (1-lambda**back) ;
```

Then we calculate the volatilities for interest rate factors:

```
mu = w # IR_trans ;
mu = j(back,1,1) * mu[+,] ;
cov_ir = (w # (IR_trans-mu))` * (IR_trans-mu) * (back/(back-1)) ;
VolGT = sqrt(vecdiag(cov_ir))` ;
mean_IR = (log(dataAll_IR[1,]) - log(dataAll_IR[&maxrow,])) / (&maxrow - 1) ;
regu = mean_IR + VolGT#VolGT/2 ;
sd_IR = 100 * dataAll_IR[1,]#exp(regu) # sqrt(exp(VolGT # VolGT) - 1) ;
```

This represents a considerable savings in lines of code compared to the original VBA program. The elegance of matrix algebra also makes the methodology more transparent. Because the spread series are first differenced and the mean is assumed to be zero, calculation is somewhat simpler:

```

nofac_SP = ncol(dataAll_SP) ;
SP_trans = dataAll_SP[1:&maxrow-1,] - dataAll_SP[2:&maxrow,] ;
mu = w # SP_trans ;
mu = j(back,1,1) * mu[+,] ;
cov_SP = (w # (SP_trans-mu))` * (SP_trans-mu) * (back/(back-1)) ;
sd_SP = sqrt(vecdiag(cov_SP))` ;

```

The interest rate and spread results are combined and labeled. The following code gives a report of the risk factor id, the current base value and the standard deviation:

```

cols_num_IR = {&risk_number_IR_num} ;
cols_num_SP = {&risk_number_SP_num} ;
cols_num_IR_SP = cols_num_IR // cols_num_SP ;
sd = sd_IR` // sd_SP` ;
dataAll_IR_SP = dataAll_IR[1,]` // dataAll_SP[1,]` ; ;
risk_factor_volatility = cols_num_IR_SP || dataAll_IR_SP || sd ;
rfv_colname ={"risk_factor_id" , "risk_factor_value" , "risk_factor_volatility"} ;

```

Below we calculate the covariance and correlation of the combined series.

```

IR_SP_trans = IR_trans || SP_trans ;
mu = w # IR_SP_trans ;
cov = (w # (IR_SP_trans-mu))` * (IR_SP_trans-mu) * (back/(back-1)) ;
nofac_IR_SP = nofac_IR + nofac_SP ;

dim = 0 ;
do i = nofac_IR_SP to 1 by -1 ;
    dim = i + dim ;
end ;
covcor = j(dim,5,.) ;
nofac_IR_SP = nofac_IR + nofac_SP ;

cor = j(nofac_IR_SP,nofac_IR_SP,1) ;
do i = 1 to nofac_IR_SP-1 ;
    do j = i+1 to nofac_IR_SP ;
        cor[i,j] = cov[i,j] / (sqrt(cov[i,i] * cov[j,j])) ;
        cor[j,i] = cor[i,j] ;
    end ;
end ;
rec = 0 ;
do i = 1 to nofac_IR_SP ;
    do j = 1 to i ;
        rec = rec + 1 ;
        covcor[rec,1] = cols_num_IR_SP[i] ;
        covcor[rec,2] = cols_num_IR_SP[j] ;
        covcor[rec,3] = cov[i,j] ;
        covcor[rec,4] = cor[i,j] ;
        covcor[rec,5] = rec ;
    end ;
end ;

```

VAR CALCULATION

As the elements of IML matrices are either all number or all character, we have to prepare the data in base SAS such that we can keep track of the variable names, etc. The names of the organization levels have to be converted into numbers. If this is a static list you can use proc format and use the number assignment to force the order of the reports or read the data into data step as in the example below:

```
data desktemp1 ;
  set sasdb.desk(keep=desk desk_sort_order process_group) ;
  where process_group = 'MVAR' ;
proc sort data = desktemp1 ;
  by desk ;
data desktemp2(drop=process_group) ;
  set desktemp1 ;
  by desk ;
  if last.desk ;
proc sort data = desktemp2 ;
  by desk_sort_order ;
data desk(rename=(desk=start) drop=process_group) ;
  retain fmtname '$deskfmt' ;
  set desktemp2 ;
  by desk notsorted ;
  if first.desk then do ;
    label + 1;
    output ;
  end ;
PROC format cntlin = desk fmtlib ;
```

The advantage of the having the data create the format is that the code doesn't have to be adjusted with each change in the database. Next we denormalized various database tables such that one matrix (mark_sql) holds risk factor and product id's and profiles, any information we'll need in to alter VaR values (currency ratio, liquidity adjustment), plus information we'll need for aggregation (business unit, asset class, etc.).

We want the report to appear as a tree structure, aggregating VaR results in a total, then at the company group level (GMAC, GM), then see each of these branch off into the various company in the groups, etc. We construct our reporting structure by merging a dataset with all the (numerical) permutations of the organization (business_group, company, etc.) with the desks in the mark_sql dataset, adding those numerical representations to create mark_val.

```
data primenum ;
  set sasdb.desk ;
  where process_group = 'MVAR' ;
  groupnum = input(put(company_group,$grpfmt.),best32.) ;
  conum = input(put(company,$compfmt.),best32.) ;
  busnum = input(put(business,$busfmt.),best32.) ;
  busunit = input(put(business_unit,$bnitfmt.),best32.) ;
  desknum = input(put(desk,$deskfmt.),best32.) ;
* proc print data = primenum ; * title 'primenum' ;
proc sort data = primenum out = prime_match ;
  by desk_id ;
data prime_mat(keep=desk_id groupnum conum busnum busunit desknum) ;
  set prime_match ;
  by desk_id ;
  if last.desk_id ;
proc sort data = sasdb.mark_sql ;
  by desk_id ;
data mark_val ;
  merge sasdb.mark_sql(in=a)
        prime_mat(in=b) ;
  by desk_id ;
  if a ;
```

We create a macro variable of product id's to use as a label for product level profit and loss matrix.

```

proc sort data = mark_val ;
  by product_id ;
data _null_ ;
%global prodcols ;
proc sql ;
select compress( tranwrd(tranwrd(tranwrd(tranwrd(upcase(tranwrd(tranwrd
( translate(left(trim(p.product_name)), '_____', '()&/-+ ' )
|| ' _ ' ||
translate(left(trim(p.product_term)), '_____', '()&/-+ ' )
|| ' _ ' ||
translate(left(trim(p.product_rating)), '_____', '()&/-+ ' )
|| ' _ ' ||
put(p.product_id,z3.0), '___', ' _ ' ), '___', ' _ ' )), '_1YR_1YR', '_1YR'),
'_3YR_3YR', '_3YR'),
'RESIDUAL_FINANCING', 'RES_FIN'),
'FIXED_RATE_LOANS', 'FIXRATLNS'))
into :prodcols separated by ','
from mark_val mv
, sasdb.product p
where p.product_id = mv.product_id ;
quit ;

```

To flatten out the tree reporting structure into something we could use in a spread sheet we remove duplicates from five files, each representing a level of the organization, retaining the numbers that will appear in the mark_val matrix and concatenating the names for the report

```
data primerow0 ;
  length primecut $ 200 ;
  set primenum(keep=groupnum company_group desk_id) ;
  primecut = left(trim(company_group)) ;

data primerow1 ;
  length primecut $ 200 ;
  set primenum(keep=groupnum company_group conum company desk_id) ;
  primecut = left(trim(company_group)) || '_' || left(trim(company)) ;

...
```

And merge the results:

```

data primerow4 ;
  length primecut $ 200 ;
  set primenum(keep=groupnum company_group conum company busnum business busunit business_unit
              desknum desk desk_id) ;
  primecut = left(trim(company_group)) || '_' || left(trim(company)) || '_' ||
             left(trim(business)) || '_' || left(trim(business_unit)) || '_' || left(trim(desk)) ;

data sasdb.primerow ;
  length primecut $ 200 ;
  set primerowt ;
  by groupnum conum busnum busunit desknum ;
  if last.desknum ;

proc contents data = sasdb.primerow out = list(keep=nobs) noprint ;
data _null_ ;
  %global coln ;
  set list ;
  nobs = nobs + 1 ;
  call symput('coln','col' || left(trim(nobs))) ; stop;

```

As SAS variable names are limited to 32 characters, we can label the levels of aggregation col1, col2, col3... We can link the label in the first row of the dataset, primerow, with col1, the label in the second row with col2, etc.

We read the covariance matrix, plus the interest rate information into IML:

```
use sasdb.mean_IR ;
  read all var{mean_IR} into mean_IR where (valuation_date = &curdat) ;

use sasdb.regu ;
  read all var{regu} into regu where (valuation_date = &curdat) ;

regu = regu` ;

use sasdb.covcor where (valuation_date = &curdat) ;
  read all var{risk_factor_id_row risk_factor_id_col cov cor} into covcor ;

dim_covcor = (-1+sqrt(1+(8*nrow(covcor)))) / 2 ;
* print dim_covcor ;

cov = j(dim_covcor,dim_covcor,.) ;
cor = j(dim_covcor,dim_covcor,.) ;
cov_cor_name = {&covcorlist} ;
rec = 0 ;
do i = 1 to dim_covcor ;
  do j = 1 to i ;
    rec = rec + 1 ;
    cov[i,j] = covcor[rec,3] ;
    cor[i,j] = covcor[rec,4] ;
    cov[j,i] = cov[i,j] ;
    cor[j,i] = cor[i,j] ;
  end ;
end ;
```

Singular Value Decomposition (SVD) is a function that allows the creation of a matrix of simulated values that has the covariance as specified. In our case &sims resolves to 15,000 (simulations):

```
call svd(u , q , v ,cov) ;
e = j(dim_covcor,&sims,100) ;
e = normal(e) ;
meanAll = j(&sims,dim_covcor,0) ;
meanAll[,1:nrow(mean_IR)] = j(&sims,1,1) * mean_IR` ;
nrq = SQRT(diag(q)) * v` ;
y = nrq` * e ;
random = y` + meanAll ;
```

Because interest rate risk factors are assumed to have non-zero means, we use saved values of mean_IR and regu to calculate our risk factor simulations:

```
random[,1:nrow(mean_IR)] = 100 * (j(&sims,1,1) * IR[1,1:nrow(mean_IR)]) #
  (exp(random[,1:nrow(mean_IR)]) - (j(&sims,1,1) * exp(regu[1,1:nrow(mean_IR)]))) ;
use sasdb.sd ;
  read all var{sd} into sd
  where (valuation_date=&curdat) ;
randomsm = random / (j(&sims,1,1) * sd`) ;
```

We subtract the base values from the profile, but concatenate these to the mark_val matrix, as we'll use these for liquidity adjustments. We create two matrices to hold the results of the module which interpolates the simulated standardized risk factors with the products profiles (note that the dimensions of the matrix are number of simulations by the number of products).

```

start intp(mv, rsm, pl, scol, ecol, m) ;
  sigma_range = ecol - scol ;
  offset = scol + (sigma_range/2) ;

  low_sigma = -1 * (sigma_range/2) ;
  high_sigma = (sigma_range/2) ;
  sigmas = low_sigma ;
  do i = low_sigma + 1 to high_sigma ;
    sigmas = sigmas || i ;
  end ;
  do i = 1 to nrow(mv) ;
    if mv[i,m] > 0 then do ;
      p = ( floor(rsm[mv[i,m]]) <> low_sigma ) >< (high_sigma -1) ) ;
      q = p + 1 ;
      d = rsm[mv[i,m]] - p ;
      l = p + offset ;
      h = l + 1 ;
      pl[i] = pl[i] +
        ((mv[i,h]- mv[i,l]) # d`) + mv[i,l])` ;
    end;
  end;
finish ;

PL_IR = j(&sims,nrow(mark_val),0) ;
PL_SP = j(&sims,nrow(mark_val),0) ;

SigmaRange = {-3 -2 -1 0 1 2 3} ;

run intp(mark_val,randomsm,PL_IR,12,18,11) ;
run intp(mark_val,randomsm ,PL_SP,4,10,3) ;

```

The interpolation module was originally written using nested loops. The interpolation process is the most time intensive of the entire program; substituting matrix notation in some parts of the module reduced the time of the program by one third. Not only is matrix notation elegant and easy to follow, it often proves to be more efficient, but it consumes more memory.

We read the file that contains the primary business categories into a matrix (primerow), dimension the matrix that will hold the totaled profit and loss aggregations by these categories:

```

use sasdb.primerow ;
read all var{groupnum conum busnum busunit desknum} into primenum ;
GMAC = j(&sims,nrow(primenum)+1,0) ;
GMAC_IR = j(&sims,nrow(primenum)+1,0) ;
GMAC_SP = j(&sims,nrow(primenum)+1,0) ;

```

The liquidity adjustment is calculated by dividing the sum of the base values of the products times the liquidity factor by the sum of the base values within each primary business category.

```
* group offset ;

do j = 1 to nrow(primenum) ;
  if (primenum[j,1] = mark_val[i,22]
    & primenum[j,2] = .
    & primenum[j,3] = .
    & primenum[j,4] = .
    & primenum[j,5] = .)
  then do ;
    GMAC[,j+1] = GMAC[,j+1] + PL[,i] ;
    GMAC_IR[,j+1] = GMAC_IR[,j+1] + PL_IR[,i] ;
    GMAC_SP[,j+1] = GMAC_SP[,j+1] + PL_SP[,i] ;
    cut_rate[1,j+1] = cut_rate[1,j+1] +
      (abs(mark_val[i,28]) * liquida) ;
    cut_rate[2,j+1] = cut_rate[2,j+1] +
      (abs(mark_val[i,28])) ;
  end ;
end ;

* company offset ;
do j = 1 to nrow(primenum) ;
  if (primenum[j,1] = mark_val[i,22]
    & primenum[j,2] = mark_val[i,23]
    & primenum[j,3] = .
    & primenum[j,4] = .
    & primenum[j,5] = .)
  then do ;
    GMAC[,j+1] = GMAC[,j+1] + PL[,i] ;
    GMAC_IR[,j+1] = GMAC_IR[,j+1] + PL_IR[,i] ;
    GMAC_SP[,j+1] = GMAC_SP[,j+1] + PL_SP[,i] ;
    cut_rate[1,j+1] = cut_rate[1,j+1] + (abs(mark_val[i,28]) * liquida) ;
    cut_rate[2,j+1] = cut_rate[2,j+1] + (abs(mark_val[i,28])) ;
  end ;
end ;
```

The profit and loss numbers are aggregated to the relevant business category by comparing the individual business level numbers in mark_val with the numbers in primenum.

Once we've aggregated all the products into their primary categories we can calculate the respective liquidity adjustment:

```
cut_rate[3,] = cut_rate[1,] / cut_rate[2,] ;
do i = 1 to ncol(cut_rate) ;
  if cut_rate[3,i] = 0
    then cut_rate[3,i] = 1 ;
end ;
```

Separate row matrices can be made for each confidence level using an IML module. These matrices can be adjusted for liquidity by dividing multiplying by the liquidity ratio:

```
fifha = fifth(GMAC) ;
gmac9995 = fifha[4,] ;
gmac999 = fifha[3,] ;
gmac990 = fifha[2,] ;
gmac950 = fifha[1,] ;

agmac9995 = gmac9995 # cut_rate[3,] ;
agmac999 = gmac999 # cut_rate[3,] ;
agmac990 = gmac990 # cut_rate[3,] ;
agmac950 = gmac950 # cut_rate[3,] ;
```

REPORTING OF VAR RESULTS

The process is repeated for each month specified and the results appended to SAS data sets. Typically we want a 12 month run to evaluate VaR over time. The results are read in IML, transposed and the relevant names added as row names.

CONCLUSION

SAS/IML is a convenient and powerful language that allows for writing customized statistics and rapid development. The improvement in performance over the original VBA model was significant: the old model required 40 minutes for 3,000 simulations for a given month; the IML model takes 10 minutes for 15,000 simulations.

REFERENCES

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J.P. Morgan's *Risk Metrics - Technical Document* available on the internet at <http://www.jpmorgan.com/RiskManagement/RiskMetrics/RiskMetrics.html>

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GMAC Two-Factor Value-at-Risk Model



