

RANKING EFFICIENCY, PROPERNESS WITH SAS PROBABILITY FUNCTIONS AND PROCEDURES

Dagan Zhang, Ph D

Program Specialist, Strategic Planning and Programs Division
Texas Workers' Compensation Commission

Abstract

Computer technology (for hardware and software) makes statistics application not only more and more popular, but also more and more effective. SAS probability functions and procedures make us much freer to use more accurate and reliable statistical concepts and algorithms while trying to improve our business administration.

For trying to improve cure efficiency of physicians and judgment properness of officers, two natural parameters for measuring them were introduced, respectively. The parameters, as random variables, have *gamma* and *binomial* distributions, respectively. For some physicians (or officers) each, their jobs construct *small samples*. To rank their job efficiency (or properness), hence, simply comparing values of the natural parameter is not a reliable way.

To rank quality (efficiency / properness, etc.) of a same kind of jobs done by a group of people individually, concept MASS, a measure in a same scale of confidence was defined in our work. And, with SAS function GAMINV, the inverse of probability function PROBGAM, MASS were directly calculated for ranking cure efficiency of physicians. With SAS probability function PROBBNML, MASS were actually calculated too for ranking judgment properness of officers. Some tricks for using these functions are needed and showed in this paper.

The data used here is illustrative only. The programs here are applicable to any OS with SAS. The paper could be regarded as a development of paper [1], however, could be read independently.

Keywords: MASS, a measure in a same scale of confidence for ranking quality (efficiency / properness / integrity etc.);
SAS probability functions PROBBNML, PROBGAM and its inverse function GAMINV.

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1. MASS, a Measure in a Same Scale of Confidence for Ranking Quality

All NTC, number of treatments / services done by a DC for a claimant, consist of a population Ω . Its corresponding random variable is denoted by Y for convenience now. Average of a sample with size n of Ω is a random variable Y_n . All values of Y_n consist of a population $\Omega(n)$. An ANTCD, average of NTC for a DC, is an item, or individual, or element of $\Omega(n)$. Denote the probability function of the sampling distribution by $F_n(y)$. Then, for $\alpha \in (0,1)$,

$$F_n(y_{n\alpha/2}) = P(Y_n \leq y_{n\alpha/2}) = 1 - \alpha/2$$

and

$$F_n(y_{n-\alpha/2}) = P(Y_n < y_{n-\alpha/2}) = \alpha/2$$

determine confidence interval

$$[y_{n-\alpha/2}, y_{n\alpha/2}]$$

with confidence probability $1-\alpha$, or significance level α .

The interval length is

$$l_{n\alpha} = y_{n\alpha/2} - y_{n-\alpha/2}.$$

To rank DC's efficiency, we can't simply declare that the greater ANTCD is, the lower efficiency is. This is because n , number of claimants cared by a DC, varies greatly with DC. Particularly, some n , sample sizes, are small. In other words, for different DC, although all ANTCD are average NTC, they base on different scales n .

To compare efficiency of all DCs, we must measure their ANTCD with different rules for different n . As a matter of fact, the less n (number of claimants cared by a DC, or sample size) is, the less stability/confidence of Y_n is since the greater the variation of Y_n is.

Namely, the less n is, the greater $y_{n\alpha/2}$ is. At same time, the less n is, the greater $l_{n\alpha}$ is. Therefore, we define a **measure in a same scale of confidence** for ranking DC's efficiency naturally as follows.

$$\text{MASS} = \frac{Y_n - y_{n\alpha/2}}{l_{n\alpha}}. \quad (1)$$

For a fixed n , the MASS is proportional to Y_n , ANTCD. So, the greater MASS is, the lower the efficiency is. In the numerate of MASS, the less n is, the greater the subtrahend is. In MASS, the less n is, the greater the denominate is. This procedure makes Y_n for all n in a same confidence scale. So, MASS, the "main part" of Y_n , can be regarded as the standard of Y_n for measuring efficiency of all DC with different n .

Remark 1. Measure MASS is so harmonious to the definition of grades of efficiency given in paper [1]! In fact,

MASS > 0 if and only if Y_n is located in the right side of the confidence interval,

MASS $\in [-1, 0]$ if and only if Y_n is located in the confidence interval,

MASS < -1 if and only if Y_n is located in the left side of the confidence interval.

We regard Y_n as "normal" in significant level α if Y_n is located in the confidence interval with confidence probability $1-\alpha$, "question" right side of the interval, "good" left side of the interval. So, Y_n is question, or normal, or good according to MASS > 0, or $\in [-1, 0]$, or < -1, respectively. If DCs are listed after sorting by descending MASS, then all question DCs are at the beginning of the list, all good DCs are at the end.

Remark 2. If distribution of Ω is normal with parameters μ and σ , as we know, the sampling distribution for n is normal with parameters μ and $\sigma / \sqrt{n}$.

For a Bernoulli distributed random variable Y whose values are 0, 1 and $P(Y=0)=p$, values of Y_n are $0/n, 1/n, \dots, n/n$. $P(Y_n = i/n)$ is probability of event: value 0 appears i times while outdrawing n elements from Ω . This means that Y_n is a random variable with binomial distribution: SAS probability function PROBBNML(p, n, i).

Generally, suppose N , number of all elements of Ω , is large enough relatively to n , say n/N is less than 2%. Samplings with and without replacement give practically identical results since an individual is very unlikely to appear more than once in a sample. (See [6].) Denote the mean and standard deviation of Ω by μ and σ , respectively. Then μ and $\sigma/\sqrt{n}$ are mean and standard deviation of $\Omega(n)$, or Y_n , respectively (factor $\sqrt{\frac{N-n}{N-1}}$ is omitted here).

If distribution of Ω is $\Gamma(\alpha, \beta)$, then the sampling distribution, distribution of $\Omega(n)$ or Y_n , is $\Gamma(n\alpha, n\beta)$ (See lemma in [1]).

Remark 3. When Ω is normal, then

$$y_{n \alpha/2} = \mu + l_{n\alpha} / 2 ,$$

and

$$l_{n\alpha} = 2z_{\alpha/2} \frac{\sigma}{\sqrt{n}} .$$

Hence,

$$\text{MASS} = \left(\frac{1}{2z_{\alpha/2} \sigma} \right) \sqrt{n} (Y_n - \mu) - \frac{1}{2} .$$

For ranking, obviously, its function is same with of

$$\text{MASS} = \sqrt{n} (Y_n - \mu) \quad (2)$$

which is same with commonly used z standard of Y_n :

$$Y_{n0} = \frac{Y_n - \mu}{\sigma / \sqrt{n}} . \quad (\text{Compare it with formula } X_0 = \frac{X - \mu}{\sigma} .)$$

Remark 4. While sampling distributions could not be determined, Chebyshev's inequality [4] could be used to calculate confidence intervals approximately under some hypotheses.

Denote the right end of confidence interval with confidence probability $1-\alpha$ by $\mu + k_n$ for Y_n . Then

$$P(|Y_n - \mu| > k_n) = \alpha .$$

By Chebyshev's inequality,

$$P(|Y_n - \mu| > k_n) \leq \frac{(\sigma / \sqrt{n})^2}{k_n^2} .$$

Suppose

$$\alpha = \frac{(\sigma / \sqrt{n})^2}{k_n^2} \quad (3)$$

i.e.

$$k_n = \frac{\sigma / \sqrt{n}}{\sqrt{\alpha}} ,$$

where μ and σ are expectation and standard deviation of Ω , respectively, μ and $\sigma/\sqrt{n}$ are expectation and standard deviation of Y_n or $\Omega(n)$, respectively.

Combining it and symmetry assumption

$$l_{n\alpha} = 2k_n, \quad (4)$$

we obtain

$$\text{MASS} = \frac{\sqrt{\alpha}}{2\sigma} \sqrt{n}(y_n - \mu) - \frac{1}{2}.$$

For ranking, the constant term and constant fact in the formula can be omitted. So,

$$\text{MASS} = \sqrt{n}(Y_n - \mu).$$

This is same with formula (2). The formula is much simpler than the definition of MASS. However, it bases on symmetry hypothesis (4). Supposition (3) makes k_n greater than or equal to $y_{n\alpha/2}$, not equal to exactly. Particularly, the harmony between MASS and grades showed in remark 1 will be missed while using the simplified formula. The not easy is traded off for uncomfortable feeling!

Remark 5. The used significance level α , as we know, is probability of making type I error. While α is low, β , the probability of making type II error is high. How to select α depends on our confidence about the approximate value taken by us for μ . As an alternative, a good way to select an approximate α for MASS might be calculating several corresponding ranked lists for different α . Compare them with each other then select one.

2. Calculating Confidence Intervals of Gamma Distribution by Function GAMINV

Suppose the inverse function of probability function $F_n(x)$ of Y_n is $G_n(p)$, then

$$y_{n-\alpha/2} = G_n(\alpha/2),$$

and

$$y_{n\alpha/2} = G_n(1-\alpha/2).$$

For gamma distribution, values of the above $F_n(x)$ and $G_n(p)$ can be calculated by SAS functions

PROBGAM

and

GAMINV,

respectively, in SAS system. See [5].

However, the SAS function PROBGAM is standard gamma distribution $\Gamma(\alpha, 1)$, not the general $\Gamma(\alpha, \beta)$ [4]. GAMINV is the inverse of PROBGAM.

To use PROBGAM and GAMINV, a transform formula between the standard and the general must be obtained. To get it, just pay attention to $F(x)=0$ for $x \leq 0$; for $x > 0$,

$$\begin{aligned} F(x) &= \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-\beta t} dt \\ &= \frac{1}{\Gamma(\alpha)} \int_0^x (\beta t)^{\alpha-1} e^{-\beta t} d\beta t \end{aligned}$$

$$= \frac{1}{\Gamma(\alpha)} \int_0^{\beta x} s^{\alpha-1} e^{-s} ds$$

$$= F_0(\beta x),$$

where $F_0(x)$ is probability function of standard gamma variable $\Gamma(\alpha, 1)$. Suppose the inverse of standard Gamma distribution $F_0(x)$ is $G_0(p)$. Since

$$F(x)=F_0(\beta x),$$

we get

$$F(x)=\text{PROBGAM}(\beta x, \alpha).$$

and

$$G(p)=G_0(p)/\beta=\text{GAMINV}(p, \alpha)/\beta.$$

Up to now, a SAS program can be written for confidence intervals about gamma sampling distributions.

3. A SAS Program for Ranking and Grading Cure Efficiency of Physicians

As indicated in [1] by PROC UNIVARIATE and PROC CHART, distribution of Ω , all NTC, is $\Gamma(\alpha, \beta)$. For the example in [1], since $\alpha/\beta=\mu=E(\text{NTC})=80.51$, and $\alpha/\beta^2=V(\text{NTC})=138.04^2=19055.042$, we get $\beta = \mu/V = 0.004225$, and $\alpha = \mu\beta = \mu^2/V = 0.34$. The sampling distribution, distribution of Y_n or $\Omega(n)$, is $\Gamma(n\alpha, n\beta)=\Gamma(0.34n, 0.004225n)$.

As we know, Greek letter α plays two roles in this paper, a parameter of gamma distribution $\Gamma(\alpha, \beta)$, and a taken significant level. We can distinguish them in the text with context naturally. In the following program, ALPHA stands for the former, AL for the latter.

```
***** Adjusting might be needed *****
%LET AL1 =0.025
%LET AL2 =0.975
* Other parameters of a MACRO statement on the bottom of the program might need to be adjusted ;
*****
LIBNAME DS 'EX4DGZ1.SAS.DATASETS';
%MACRO DTYPE(DR, FYIJ, DEST, MU, V);
DATA W;
  SET DS.DLST0;
  WHERE DRTYPE="&DR";
  ANTCD=NV / NCLM;
  /* NV is number of treats / services of a physician; NCLM is number of injured workers cared by
  the physician. */
  BETA=NCLM*&MU / &V;
  ALPHA=&MU*BETA;
  XU=GAMINV(&AL2, ALPHA) / BETA;
  XL=GAMINV(&AL1, ALPHA) / BETA;
  MASS=(ANTCD-XU) / (XU-XL);
  GRADE='Normal';
  IF MASS < -1 THEN GRADE='Good';
  IF MASS > 0 THEN GRADE='Question';
PROC SORT;
  BY DESCENDING MASS NCLM;
```

```

PROC REPORT DATA=W SPLIT="*" CENTER;
TITLE1
  "RANKING AND GRADING &DR BY AVERAGE NUMBER OF TREATMENTS / SERVICES
  FOR CLAIMANTS OF A &DR";
TITLE2 "FOR CLAIMS INJURED IN FISCAL YEAR &FYIJ";
TITLE3 "WITH MEDICAL FORMS ESTABLISHED BY JUN 8, &CYEST";
TITLE4 "=====";
COLUMN PHYSLIC NCLM ANTCD GRADE;
DEFINE PHYSLIC / DISPLAY FORMAT=$8. WIDTH=8 "DOCTOR*LICENSE";
DEFINE NCLM / SUM FORMAT=COMMA9.0 WIDTH=9 "NUMBER OF*CLAIMANTS";
DEFINE ANTCD / COMPUTED FORMAT=COMMA9.2 WIDTH=16
  "AVERAGE # OF T/S*OF CLAIMANTS";
DEFINE GRADE/DISPLAY FORMAT=$8. WIDTH=8 "GRADE";
RUN;
%MEND;
%DTYPE(DC, 2000, 1Jun2004, 80.51, 19055.042);
/* MU and V are mean & variance of population NTC */
RUN;

```

Illustration Output: List of All DCs Sorted by Descending MASS

DOCTOR LICENSE	NUMBER OF CLAIMANTS	AVERAGE # OF T/S OF CLAIMANTS	GRADE
C11119	63	391.22	Question
C12008	84	285.93	Question
C64132	17	514.53	Question
C14379	276	178.99	Question
C24701	13	532.08	Question
...	...	...	...
C02436	30	136.97	Question
C32689	15	163.87	Normal
C11223	2	347.00	Normal
C13598	4	4.75	Normal
...	...	...	...
C10001	65	50.29	Good
C20090	138	55.85	Good
C10203	551	63.45	Good

4. Ranking Judgment Properness of Officers

A decision against / for a claimant for an issue made by a hearing officer in a contested case hearing at TWCC could be appealed by a party. An appeals panel may affirm / reverse the decision.

Suppose that CA, CF are numbers of all issues with decisions against, for claimants, respectively, made by an officer. CAF is sum of CA and CF. CAAF and CFAA are numbers of the issues with decision against and for, respectively, made by the officer and the reversing made by appeals panels. AR is sum of CAAF and CFAA. Then rate $P=AR/CAF$ is used to measure judgment properness of the officer.

Since numbers CAF vary with officers and some of them are small, to rank officers, we can't simply compare their P only. By Ω denote all issues with decision against / for made by officers and decision affirming / reversing made by appeals panels. An event in Ω gets value $Y=0$ if the decision made by the appeals panel is reversing. $P(Y=0)=p$ is close to quotient: number of all reversed over number of all issues with decision against / for made by officers. Otherwise, value $Y=1$. Y could be regarded as a random variable with Bernoulli distribution. Then $Y_n (=P)$ is a binomial sampling distribution $B(p, n, i)$, where n is number of all issues judged by the officer. Hence, we calculate

$$MASS = \frac{Y_n - y_{n\alpha/2}}{l_{n\alpha}} = \frac{\frac{AR}{CAF} - \frac{b_2 + 0.001}{CAF}}{\frac{b_2 + 0.001}{CAF} - \frac{b_1 - 0.001}{CAF}} = \frac{AR - b_2 - 0.001}{b_2 - b_1 + 0.002},$$

Where

$$b_1 = \underset{B(p, n, i) \geq \alpha/2}{Min} \{i \mid i = 0, 1, \dots, n\},$$

$$b_2 = \underset{B(p, n, i) \geq 1 - \alpha/2}{Min} \{i \mid i = 0, 1, \dots, n\}.$$

Based on the formulas, a program section for b_1 , b_2 and MASS is as follows.

```
***** Adjust significance level *****;
%LET ALPHA = 0.02;
*****;

.....
PROC SUMMARY DATA=A;
  VAR CAF AR;
  OUTPUT OUT=S SUM=SCAF SAR;
DATA _NULL_;
  SET S;
  p=SAR/SCAF;
  CALL SYMPUT('p', p);
DATA B;
  SET A;
  DO I = 0 TO CAF;
    PB = PROBBNML(&p, CAF, I);
    B2 = I;
    IF PB GE 1-&ALPHA/2 THEN GO TO F;
  END;
F: DO I = 0 TO CAF;
  PB = PROBBNML(&p, CAF, I);
  B1 = I;
  IF PB GE &ALPHA/2 THEN GO TO G;
END;
G: MASS = (AR-B2-0.001) / (B2-B1+0.002);
.....
```

REFERENCES

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