

Truncated Levy walks applied to the study of the behavior of Market Indices

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This work is devoted to the study of statistical properties of Major Market Indices and Emergent Market Indices. We conclude that the behavior of the return is compatible with a slow convergence to a gaussian distribution.

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1. Introduction

In recent years there has been a growing literature in financial economics that analyzes the major Stock indices in developed countries [1-5].

At the same time a new discipline: Econophysics, has been developed [6]. This discipline studies the application of mathematical tools that are usually applied to physical models, to the study of financial models. Simultaneously, there has been a growing literature in financial economics analyzing the behavior of major Stock Indices [7-11].

The Statistical Mechanics theory, like phase transitions and critical phenomena have been applied by many authors to the study of the speculative bubbles preceding a financial crash (see for example [12], [13]). In these works the main assumption is the existence of log-periodic oscillation in the data. The scale invariance in the behavior of financial indices near a crash has been studied in [14-15].

The statistical properties of the temporal series analyzing the evolution of the different markets have been of a great importance in the study of financial markets.

The empirical characterization of stochastic processes usually requires the study of temporal correlations and the determination of asymptotic probability density distributions

(pdf). The first model that describes the evolution of option prices is the Brownian motion. This model assumes that the increment of the logarithm prices follows a diffusive process with Gaussian distribution [16]. However, the empirical study of temporal series of some of the most important indices shows that in short time intervals the associated **pdf** have greater kurtosis than a gaussian distribution [5], and the Brownian motion does not describe accurately the evolution of financial indices near a crash. The first step in order to explain this behavior was done in 1963 by Mandelbrot [17]. He developed a model for the evolution of the cotton price by a stable stochastic non gaussian Levy process [18]. However, these distributions are not appropriated for working in long-range correlation scales. These problems can be avoided considering that the temporal evolution of financial markets is described by a Truncated Levy Flight (**TLF**) [19-20].

Most of the studies mentioned before have been done with financial indices of developed markets that have a great volume of transactions, and only recently these studies have been extended to emergent markets [21-22].

This work is devoted to the study of Major market indices as well as emergent market indices.

The presentation is organized as follows: In the first section we give a short introduction to the Levy distributions. In the second section we present the indices that will be analyzed and we give a relation between them and the TLF. Finally, we conclude that our results are compatible with a power law, and also with a TLF distribution. The numerical analysis has been done by using SAS and Matlab. In [21] we also detected the presence of long-range correlations in several financial indices corresponding to emergent markets.

2. The truncated Levy flight

Levy [23] and Khintchine [24] solved the problem of the determination of the functional form that all the stable distributions must follow. They found that the most general representation is through the characteristic functions $\varphi(q)$, by the following equation:

$$\ln(\varphi(q)) = \begin{cases} i\mu q - \gamma|q|^2 \left[1 - i\beta \frac{q}{|q|} \tan\left(\frac{\pi}{2}\alpha\right) \right] & (\alpha \neq 1) \\ \mu q - \gamma|q| \left[1 + i\beta \frac{q}{|q|} \frac{2}{\pi} \ln|q| \right] & (\alpha = 1) \end{cases}$$

where $0 < \alpha \leq 2$, γ is a positive scale factor, μ is a real number and β is an asymmetry parameter that takes values in the interval $[-1,1]$.

The analytic form for a stable Levy distribution is known only in the cases:

$\alpha = 1/2$, $\beta = 1$ (Levy-Smirnov distribution)

$\alpha = 1$, $\beta = 0$ (Lorentz distribution)

$\alpha = 2$, (Gaussian distribution)

We consider symmetric distributions ($\beta = 0$) with zero mean value ($\mu = 0$). In this case the characteristic function takes the form:

$$\varphi(q) = e^{-\gamma|q|^\alpha}$$

As the characteristic function of a distribution is its Fourier transform, the stable distribution of index α and scale factor γ is

$$P_L(x) \equiv \frac{1}{\pi} \int_0^\infty e^{-\gamma|q|^\alpha} \cos(qx) dq$$

the asymptotic behavior of the distribution for big values of the absolute value of x is given by:

$$P_L(|x|) \approx \frac{\gamma \Gamma(1+\alpha) \sin(\pi\alpha/2)}{\pi |x|^{1+\alpha}} \approx |x|^{-(1+\alpha)}$$

and the value in zero $P_L(x=0)$ by:

$$P_L(x=0) = \frac{\Gamma(1/\alpha)}{\pi \alpha \gamma^{1/\alpha}}$$

The fact that the asymptotic behavior for big values of x is a power law has as a

consequence that the stable Levy processes have infinite variance. In order to avoid the problems arising in the infinite second moment *Mantegna et al* considered a stochastic process with finite variance that follows scale relations called *Truncated Levy flight (TLF)* [19]. The **TLF** distribution is defined by:

$$P(x) = \begin{cases} 0 & x > l \\ cP_1(x) & -l < x < l \\ 0 & x < -l \end{cases}$$

where $P_1(x)$ is a symmetric Levy distribution and c is a normalization constant. The only stable distributions are the Levy distributions, the **TLF** distribution is not stable, but it has finite variance, so it converges to the Gaussian distribution, but unlike another **pdf**, its convergence is very slow. Another property of the **TLF** is that the cut that it presents in its tails is very abrupt. Koponen [20] considered a **TLF** in which the cut function is a decreasing exponential characterized by a parameter l . The characteristic function of this distribution is defined as:

$$\varphi(q) = \exp \left\{ c_0 - c_1 \frac{(q^2 + 1/l^2)^{\alpha/2}}{\cos(\pi\alpha/2)} \cos[\alpha \arctan(l|q|)] \right\}$$

with c_1 a scale factor

$$c_1 = \frac{2\pi \cos(\pi\alpha/2)}{\alpha \Gamma(\alpha) \sin(\pi\alpha)} A t$$

and

$$c_0 = \frac{l^{-\alpha}}{\cos(\pi\alpha/2)} c_1 = \frac{2\pi}{\alpha \Gamma(\alpha) \sin(\pi\alpha)} A l^{-\alpha} t$$

3. Methods and data analysis

We studied several market indices using data of four countries (daily close values): from July 1st 1997 to June 3th 2005; Argentine (MERVAL), Brazil (BOVESPA), India (BSE 30). The number of data for the three indices is 1962. We also used data of the SP500, an index of the New York Stock Exchange. In the last case, the data correspond to two

different periods: the first one from April 14th, 1950 to July 15th, 2002 (13145 points) and the second one with the same period of the first three indices. The reason of this selection is to compare the results for developed economies with the results for emergent markets. For this reason, we considered two periods: one short period, equal to the periods corresponding to the indices from emergent economies; and a longer one, in order to check the consequences of working with a small number of points.

In this work, the analyzed stochastic variable is the return (r_t), defined as the difference of the logarithm of two consecutive indices prices:

$$G_t = \log(I_t) - \log(I_{t-T})$$

where T is the difference (in labor days) between two values of the index. In order to compare the returns for different values of T we define the normalized return as:

$$g \equiv \frac{G - \langle G \rangle_T}{\sigma}$$

where $\sigma^2 \equiv \langle G^2 \rangle_T - \langle G \rangle_T^2$, and $\langle \dots \rangle_T$ denotes the arithmetic average over the series of temporal scale T . The empirical study of temporal series of some of the most important indexes shows that in short time intervals the associated **pdf** have greater kurtosis than a gaussian distribution.

In Figure 1 we show the accumulated probability distribution of the four shorter indices, and in Figure 2 the normalized returns of the versions of the SP500 index. These graphics portrait an asymptotic behavior corresponding to a power law:

$$P(g > x) \approx \frac{1}{x^\alpha}$$

In table 1 we give the exponents α calculated for each index, all the values are strictly greater than 2. These values are greater than the values corresponding to a Levy distribution; however, Weron [25] concluded that these values could be a consequence of working with finite samples. The behavior is compatible with a slow convergence to a gaussian distribution. We can verify this statement by studying the momentum evolution. The momentum μ_k is defined by:

$$\mu_k = \langle |g|^k \rangle_T$$

The numerical analysis was performed with different values of T

In Figure 3, corresponding al index SP500 with more data, we can observe that the tendency to diverge in the moments of grater order is much more pronounced that for the second moment. This behavior is less pronounced in the graph corresponding to SP500 (Figure 4) with less data point.

These graphs correspond to the results of Gopikrishnan et al [26], and to the results for Latin American Indices in [21].

The numerical analysis has been done by using SAS and Matlab.

4. Conclusions

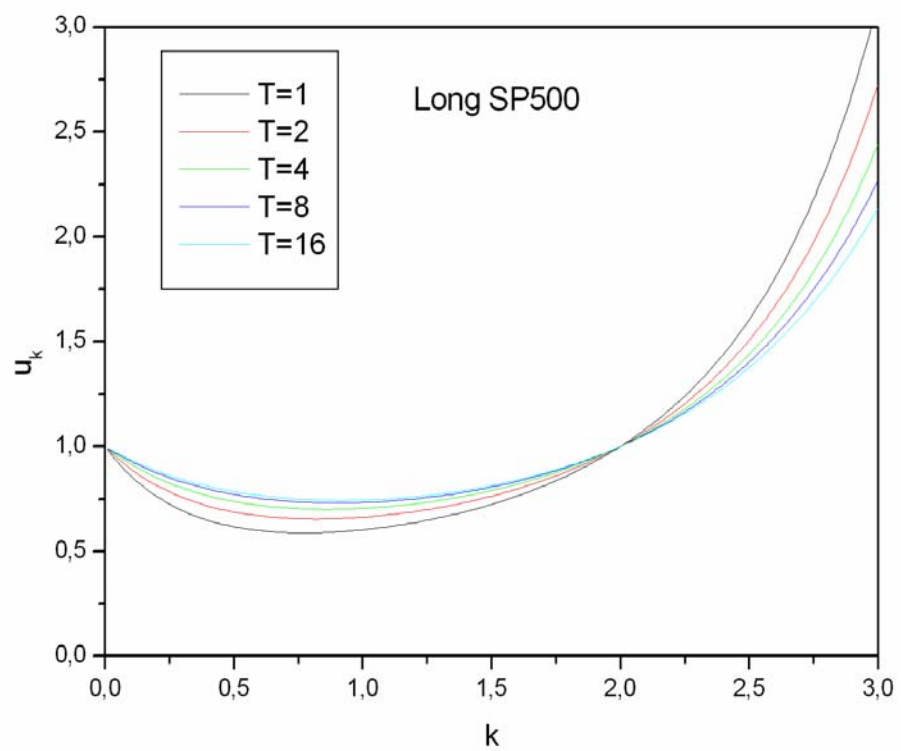
In this work we did an empirical study of the statistical behavior of four different financial indices: the SP500 belongs to a well developed economy and the other three financial indices belonging to less developed economies. Although we obtained compatible results with a potency law, it is not possible to conclude that the statistical distribution is not a TLF. This fact is because only when working with very large data sets (greater than 10^6 observations) is possible to detect a power behavior, allowing an estimation of the exponent α . Therefore, it is not possible to conclude that financial indices corresponding to emergent economies have a substantially different behavior than the financial indices corresponding to well developed economies.

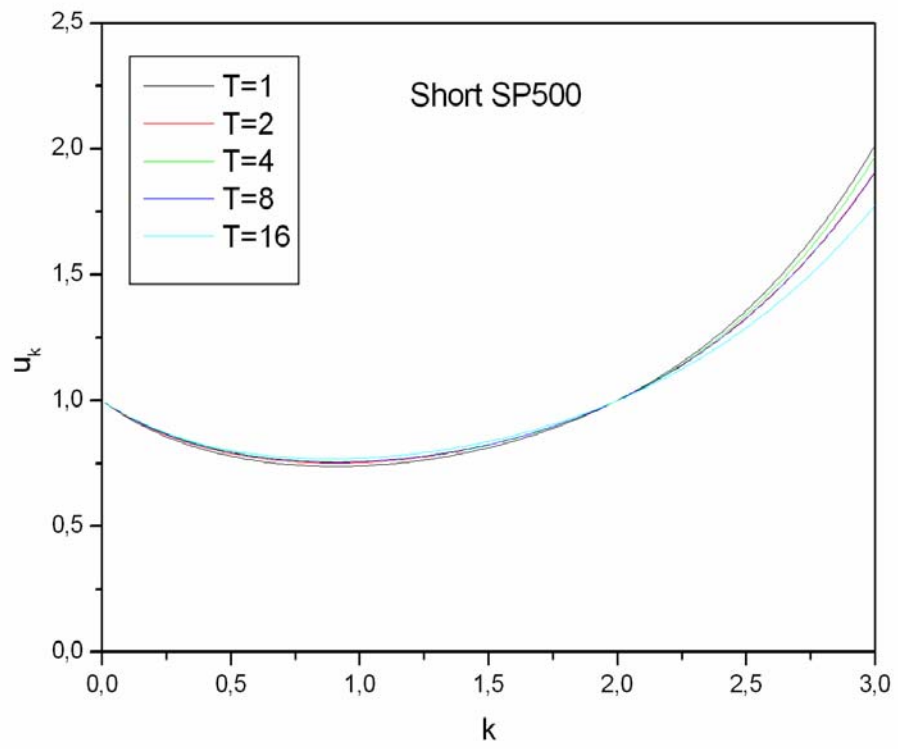
An explanation for this fact is that even if in the emergent markets the volume of transactions is small, these markets are more sensitive to the behavior of individual investors and traders, and the strategies that these investors use are similar to the strategies used by investors in developed markets, with a bigger volume of transactions.

Table 1: Values of the exponent α for the different indices.

SP500	LongSP500	BOVESPA	MERVAL	BSE 30
2.88	3.29	2.60	2.31	3.18

Figures 3,4





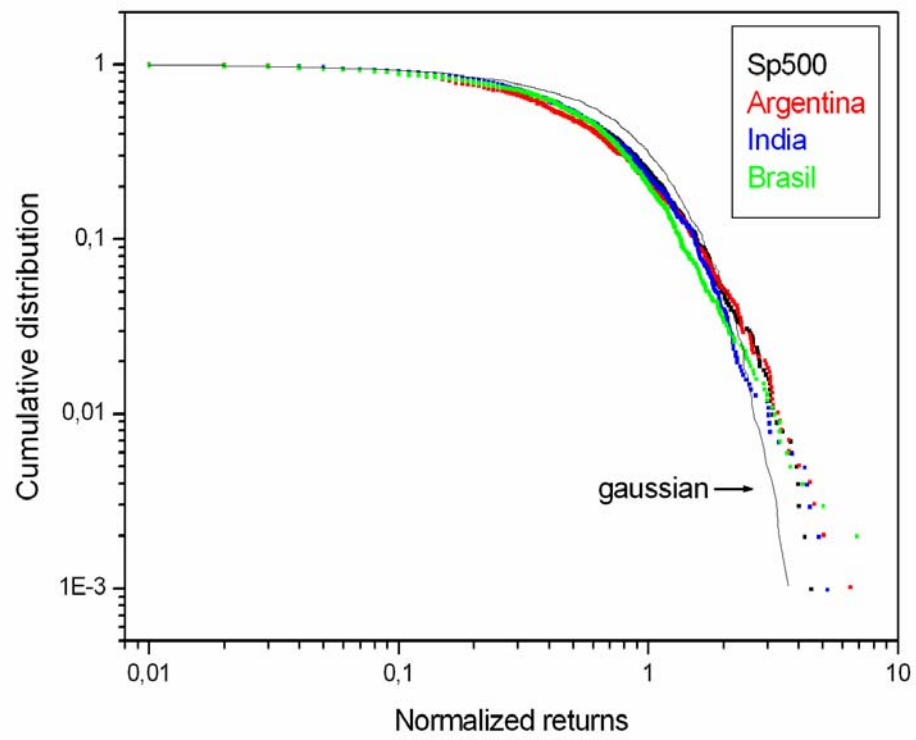


Figure 1

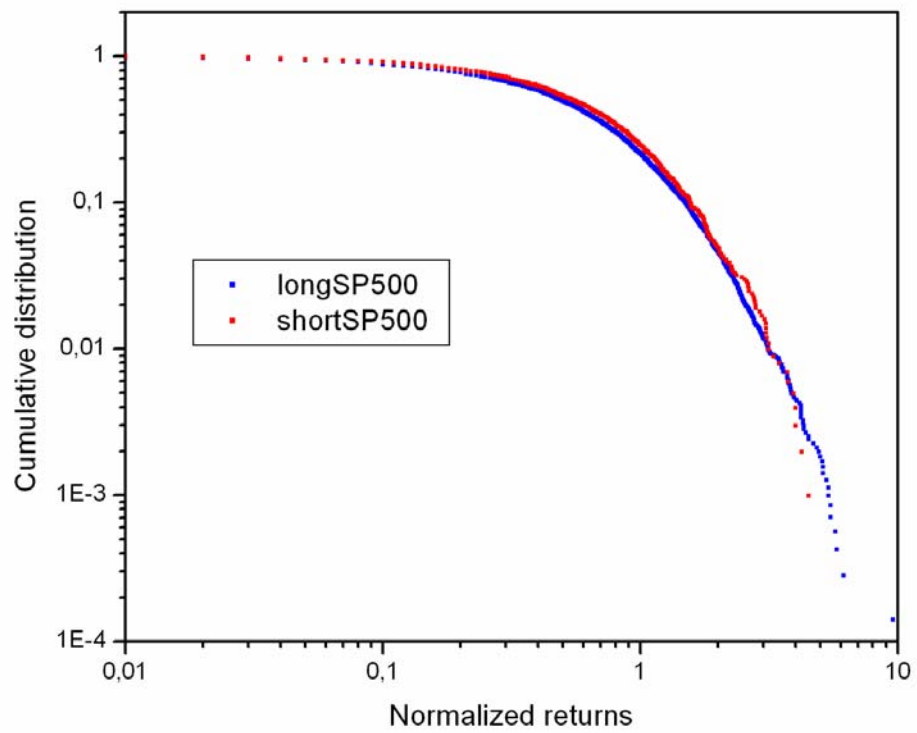


Figure 2

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