

MrPD - A SAS® Software Application Using Desirability Functions For Optimization of Multiple Responses

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Abstract

MrPD (Multi-property Robust Product Design) is a SAS software application that implements the desirability function technology of Derringer and Suich¹ for multiple response optimization. Using the powerful capabilities of SAS® Screen Control Language, the application leads the product developer through the optimization process to find the product configuration that provides the best balance of response values. MrPD combines regression models fit to individual responses with desirability functions to produce an overall desirability surface, with strong graphic capabilities to provide an intuitive feel for results. The maximum for overall desirability locates the optimum configuration. Extra features include capabilities to save regression models and desirability functions fitted, and the assignment of weights to responses to explore the impact of individual responses on the optimum configuration. MrPD is extremely flexible and can be applied to any general product development problem of identifying the product configuration with the best overall performance. The technology and application are discussed and illustrated with an example.

Introduction

An important problem to be solved in businesses everywhere is the product design problem. The issue is "How can I structure my production process so that the resulting product best meets customer needs?" One of the most difficult aspects of this problem is to determine which of many performance criteria is most important. Is sale price the most important criterion? Is it durability? Is it time of service before maintenance is required? Is it color, or style?

Actually, it is rare that only one criterion is essential. Generally, we expect products we buy (whether tires, concrete, suits, cookies, or insurance policies) to perform well on many criteria. Thus, after collecting data on the performance of several prototype products (from a designed experiment,

hopefully), product designers try to evaluate the effects of the test variables upon each response. However the problem of how to select the prototype that best meets overall performance requirements remains. Generally, the best product in one particular performance dimension will be sub-optimal by some other criterion. Thus, the product designer is left with optimizing one or two critical performance measures and hoping the rest are "not too bad". Another approach is calculating a weighted average of all the performance measures, then modeling and optimizing the weighted average.

This paper will discuss a technology that uses desirability functions to capture whether a particular value for a performance measure is unacceptable, mediocre, good or excellent, on a 0 to 1 scale. With this input from the product designer, and weights assigned for each performance measure, the product with the greatest overall desirability can be identified. This product will be the product with the best balance of performance measures. This technology, and a SAS software application that implements it (MrPD), will be discussed and illustrated with an example.

Using Desirability Functions For Optimization of Multiple Responses

In 1980, Derringer and Suich described a technology for multiple response optimization using desirability functions. Recognizing that the product configuration that will yield the best values for one performance measure is likely to be sub-optimal for other performance measures, this technology balances these judgments objectively through the use of *desirability functions*. The desirability function relates values for each response to how "desirable" that value is. Desirability values range from zero to 1, with zero indicating an unacceptable level for the response, and 1 indicating that no more acceptable value is possible. Some example desirability functions are given in Figure 1.

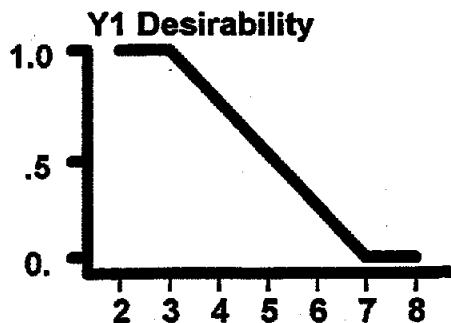


Figure 1a Lower Values are Best

Figure 1a shows the desirability function for response variable Y_1 . This is a "lower values are best" type desirability function. For Y_1 , values of 3 or less are excellent, with 2 being no improvement over 3. However, the desirability decreases as the response values increase up to a level of 7, beyond which they are unacceptable. Thus, a product with a value of 6.5 for this response would be barely acceptable, but one with a value of 7.5 would be unacceptable. This type of desirability function might be appropriate for responses like product cost, emission yields, or even a bitter taste attribute for cough syrup.

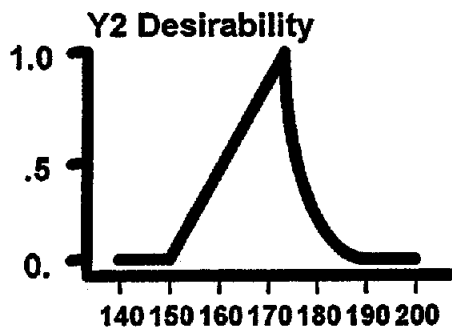


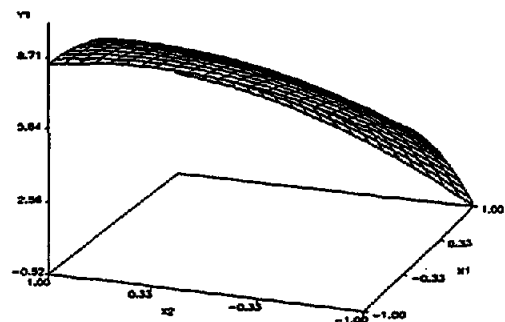
Figure 1b Middle Values are Best

Figure 1b shows the desirability function for response variable Y_2 . This is a "middle values are best" type desirability function. For this response, values below 150 or above 190 are unacceptable, while 175 is the best value. As the value moves away from the target, the desirability decreases, with greater sensitivity above target. The curvature of the desirability function shows the sensitivity to changes from the best values. Figure 1b indicates that symmetry is not required. This type of desirability function might be appropriate for a sweetness attribute for a candy bar, size of a part to be used in further assembly, or hardness of a steel support beam.

A "higher values are best" desirability function form is also available (similar to Figure 1a, but with increasing desirability as the response value

increases). These three functional forms allow great flexibility for capturing the relationship between performance measures and product desirability. It is important to recognize that the desirability function is provided by the product designer, independent of any data collection. It describes what values would make the product most appealing, without regard to any products already made.

Data from product design experiments are used to fit regression models to predict each response based on the product configuration. With these models, contour plots or 3D plots of the regression surface can be developed. Then, by mapping each point on the regression surface through the desirability function, a surface of predicted desirability of that response can be generated. This has the effect of rescaling each response variable from 0 to 1 (or whatever range of desirabilities within that interval is represented by the predicted values). It is easier to identify promising product configurations using contour or 3D plots of these desirability surfaces since large values (as close to 1 as possible) are best, rather than having to read some intermediate values. Figure 2a shows the predicted regression surface for variable Y_1 referenced in Figure 1a. It shows that values for Y_1 are largest for small values of X_1 (in the front of the 3-D plot), with values decreasing as X_1 increases, especially for small values for X_2 (in the back right corner).

Figure 2a Regression Surface for Y_1

The desirability surface resulting from applying the desirability function for Y_1 to the regression surface in Figure 2a is shown in Figure 2b. You will recall that the desirability function for Y_1 was of the "lower values are best" type. This is shown in the desirability surface with a plateau in the back right corner (large values for X_1 , small values for X_2) where the predicted values are 3 or less, which are most desirable and equally good according to the desirability function (see Figure 1a).

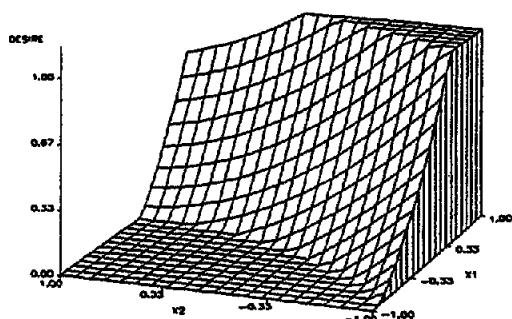


Figure 2b Desirability Surface for Y1

This example shows how desirability functions provide "standardized" values for each product performance measurement. This allows desirability values to be weighted for relative importance and combined to calculate "overall desirability" using the weighted geometric mean of the individual values. The geometric mean of desirability (D) is calculated from the individual desirability values (D_i) as follows:

$$D = \left(\prod_{i=1}^n D_i^{w_i} \right)^{\frac{1}{\sum w_i}}$$

The geometric mean is used because it guarantees that if a given product configuration has an unacceptable value for any performance measure (zero desirability), it will be judged to be unacceptable overall. In practice, this calculation can be performed easily using logs.

To complete the process, the space of possible product formulations can be searched to maximize the overall desirability. The product formulation with the highest overall desirability is the one with the best balance of properties. In addition, viewing contour or 3D plots of the overall desirability surface permits the product designer to assess the sensitivity of product performance at the best product configuration for robustness evaluation.

MrPD - A SAS Application

MrPD (Multi-property Robust Product Design) is a SAS application that implements desirability function technology and optimization using SAS Screen Control Language. It is intended for use by product designers who are not SAS experts. Starting with a SAS data set of experimental results, the product designer identifies the design variables and the response variables. From there, the product designer proceeds through the menu to:

1) Fit Regression Models to the Data

Up to full second order models can be fit, with a different model for each response variable if desired. Alternatively, the same model can be fit to all responses in one step. Regression surfaces can be viewed (contour or 3D plots) with total user control over the axis variables and levels of control variables. Plots can be viewed on screen or sent to an output file for later routing to a plotter or printer.

2) Specify Desirability Functions

Desirability functions of the forms "lower values are best", "middle values are best", or "higher values are best" can be selected. In addition, coefficients for curvature may be input to indicate sensitivity to being off target. Also, the "box" desirability function form (all values within specified limits are equally acceptable, but values outside them are not) can be selected if appropriate. This desirability function form, unique to MrPD, allows constrained searches (of which mixture experiments would be special cases). After specification of each desirability function, it is displayed on screen, and can be routed to a plotter or printer.

3) Calculate Overall Desirability and Optimize

The product designer specifies weights for each response and can view (or write to a graphics output file) contour or 3D plots of overall desirability, or select 'Optimize' to find the product configuration within the range of design variables in the original data set with the highest overall desirability. The search of the design space is done using a Hooke and Jeeves² grid search technique. MrPD presents the optimum configuration with predicted values for each response and the individual desirabilities of those values at the optimum.

MrPD is very flexible in its capabilities for graphical display, desirability function specification, and weights for responses. It is easy to use in that input is facilitated by selection lists and choice groups, with help available for each screen displayed. Its interactive nature allows the product designer to look at the results in many different ways and it provides a comfortable, intuitive feel for the effects of product configuration changes. All the graphs shown in this paper were produced using MrPD.

In practice it is useful to run the optimization routine several times using different values for the weights. In particular, weights for some responses can be set to zero. This allows the product designer to find the best possible values for one or more responses when not constrained with finding acceptable values for all responses. This provides a realistic

framework for further optimization work. After an optimum is identified, if the predicted value for one or two responses is not as good as the product designer was hoping for, the weights can be adjusted and the optimization repeated. Since the overall desirability surface can have local maxima, it is a good idea to start the optimization at different points in the space. Also, since the search region is bounded by the range of observed data, it is a good idea to look at graphical displays of the overall desirability surface at the optimum, to make sure that the search procedure did not get hung up at the edge of the space or a point of discontinuity when "box" type desirability functions are used.

Depending on the size of the problem, the entire process can be conducted in a few hours, compared to days of agonizing over tradeoffs without such a tool. This is an incredibly powerful and efficient way to solve the product design problem.

Some may argue that MrPD requires too much subjective input; i.e., desirability functions and their associated weights. Of course, there is some subjectivity. However, the proof of its effectiveness is in the predicted responses for the optimum configuration. If all responses have good (high desirability) values, then how that configuration was identified is really irrelevant.

Example

The following example was fabricated but illustrates the concepts of desirability functions, desirability surfaces, and overall desirability. It deals with identifying optimum baking conditions for a breakfast cereal. The design variables for the example are baking temperature, baking time, and percent sugar in the recipe. The performance measures to be evaluated are flake size, crispness, and sweetness. While crispness and sweetness are sensory attributes, flake size is determined analytically. This illustrates that performance measures of any aspect of the product can be included in these analyses, whether strength, brightness, taste, or even cost or variability of strength. This technology can handle them all, and find a solution to balance requirements for performance measures of all types.

The design used was a central composite design³ with axial points 1 design unit from the center and five center points. Table 1 lists the data and Figure 3 shows a schematic of the design.

Table 1 Data for Cereal Example

Obs.	Temp	Time	Sugar	Size	Crisp	Sweet
1	-1	-1	-1	15.29	3.30	4.96
2	-1	-1	1	16.59	1.97	6.01
3	-1	1	-1	8.06	3.52	4.02
4	-1	1	1	7.22	2.79	7.15
5	1	-1	-1	9.89	3.96	3.42
6	1	-1	1	11.90	3.82	3.66
7	1	1	-1	5.31	5.32	5.30
8	1	1	1	4.46	5.22	6.93
9	-1	0	0	12.83	3.81	6.66
10	1	0	0	11.05	5.06	5.80
11	0	1	0	14.80	3.90	5.87
12	0	-1	0	7.11	4.86	6.39
13	0	0	-1	12.19	4.56	4.29
14	0	0	1	12.20	4.13	6.70
15	0	0	0	10.92	4.30	6.23
16	0	0	0	10.40	5.11	6.13
17	0	0	0	11.31	5.01	5.93
18	0	0	0	12.36	4.66	6.66
19	0	0	0	12.60	4.88	6.28

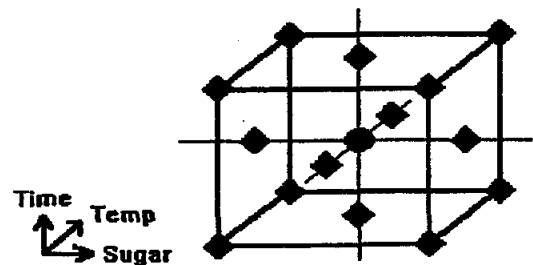


Figure 3 Central Composite Design With Axial Points on the Cube Face

Thus, the product development problem is to identify the levels of baking temperature, baking time, and sugar that will provide the best balance of product measures crispness, flake size, and sweetness.

The desirability functions for this example (see Figure 4a-c below) show that crispness must be at least 4, with no additional benefit for crispness above 7; flake size must be bigger than 5, with no additional benefit for flake size bigger than 15; and sweetness values of 6 are ideal, while values from 4 to 8 are acceptable. Note that the curvature from 4 to 6 for the sweetness desirability function indicates a greater sensitivity to being off target on the low side.

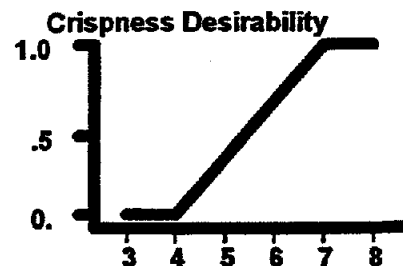


Figure 4a Crispness Desirability

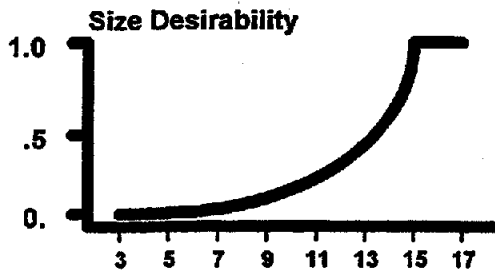


Figure 4b Size Desirability

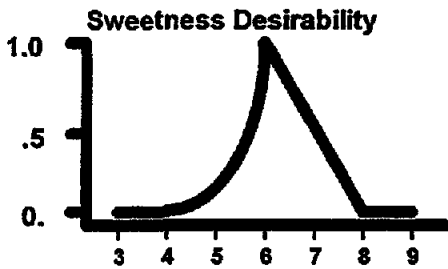


Figure 4c Sweetness Desirability

Regression models fit to the data are displayed in Figures 5a-5c. These surfaces show the effects of varying baking time and temperature with sugar held constant at 1. Of course, to get a more complete understanding of the behavior of the response variables to process changes, you would need to look at other levels of sugar as well.

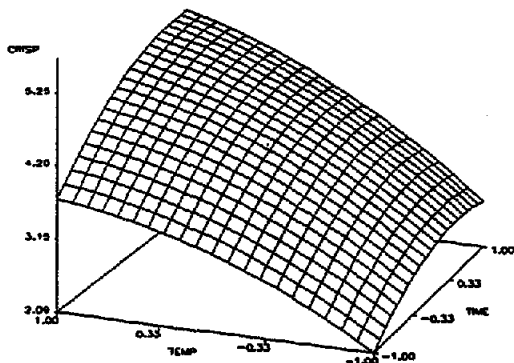


Figure 5a Regression Surface for Crispness, Sugar = 1

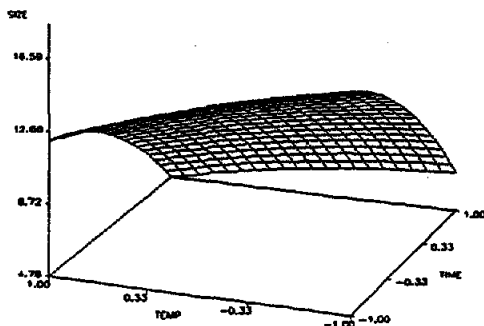


Figure 5b Regression Surface for Size, Sugar = 1

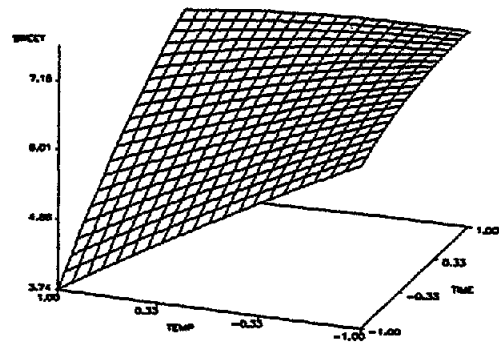


Figure 5c Regression Surface for Sweetness, Sugar = 1

As before, the desirability surfaces for the response variables can be developed by mapping the predicted response values through their respective desirability functions. These are shown in Figures 6a-6c, again displaying the effects of varying time and temperature with sugar held constant at 1. Close review of the desirability functions and regression surfaces helps in understanding the appearance of the desirability functions. For size, a contour plot is displayed rather than a 3-D plot, illustrating this capability of MrPD.

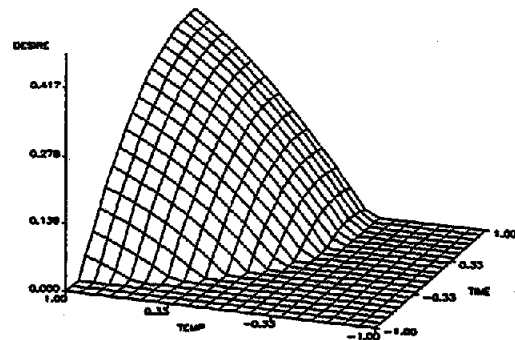


Figure 6a Desirability Surface for Crispness, Sugar = 1

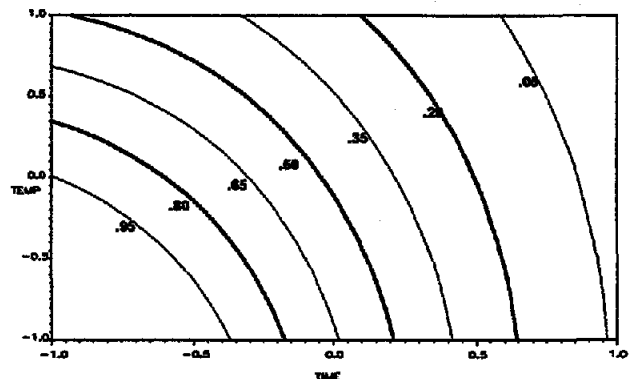


Figure 6b Desirability Contours for Size, Sugar = 1

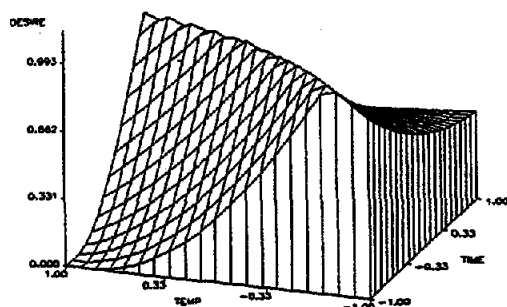


Figure 6c Desirability Surface for Sweetness, Sugar = 1

Note that the desirability functions for crispness and size are in conflict with each other. Better crispness will be obtained at higher baking times and temperatures, but larger flakes are produced at lower baking times and temperatures. Thus, no solution where both these responses are close to their best values is feasible. The overall desirability surface (with equal weights for crispness, size, and sweetness) shown in Figure 7 below indicates how these surfaces fit together, again, with sugar held constant at 1.

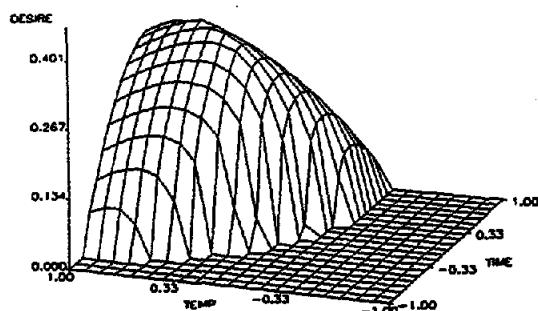


Figure 7 Overall Desirability Surface, Sugar = 1
Equal Weights for Crispness, Size, and Sweetness

The overall desirability surface shows that for conditions where crispness was unacceptable (desirability was zero) the overall desirability is also zero, thus eliminating those conditions from consideration. The highest overall desirability for sugar = 1 appears to be about .6 for temperature and .3 for time. The optimum found by MrPD (and indicated in Figure 8 below) is at temperature = .30, time = -.10, and sugar = -.10.

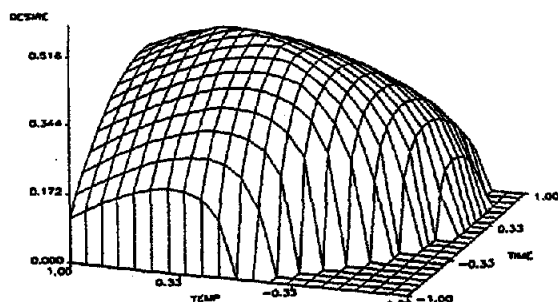


Figure 8 Overall Desirability Surface, Sugar = -.1
Equal Weights for Crispness, Size, and Sweetness

For these process settings, predicted crispness is 4.9 with a desirability of .31, size is 11.6 with a desirability of .44, and sweetness is 6, with a desirability of 1.0. Overall desirability at the optimum is .52. Of course, if the product designer feels he could "give up" some desirability on sweetness, he could run the optimization again, with the weight for sweetness less than those for size and crispness (even as low as zero) to see how much better he could do on size and crispness performance. MrPD permits different weighting scenarios through a single screen that provides an efficient, intuitive perspective on the system behavior.

Conclusions

In conclusion, the use of desirability functions, as implemented in MrPD, provides an extremely efficient way for a product designer to identify the product configuration with the best balance of a wide range of performance measures. As the number of performance measures and test variables increase, MrPD is even more useful, for it is unlikely the product designer will identify as good a solution. If the product proposed by MrPD matches up favorably on the performance measures, then it is unimportant how it was identified. That is, even though some subjectivity is involved in the process, MrPD is an efficient tool for solving the multiple response optimization problem.

References

- ¹Derringer, George and Suich, Ronald, "Simultaneous Optimization of Several Response Variables", *Journal of Quality Technology*, Vol. 12, No. 4, 1980, pp. 214-219.
- ²Hooke, R. and Jeeves, T. A., *Journal of the Association of Computing Machinery*, Vol. 8, No. 2, 1962, p.212.
- ³Cochran, William G., and Cox, Gertrude M., *Experimental Designs*, 2nd Edition, John Wiley and Sons, 1957, p. 343.

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