

## Contour Profiling to Optimize Multiple Responses Across Multiple Factors

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When experiments are done with several responses measured, it is not obvious what to optimize. Rarely can you find factor combinations that lead to optimizing all the responses simultaneously—there is usually a trade off.

A classic example is the tire tread experiment from Derringer and Suich (*Journal of Quality Technology*, 1980, 214-219). There were three factors in the experiment, hydrated SILICA level, SILANE coupling level, and SULFUR level, arranged in a central composite design. Four responses were measured, ABRASION, MODULUS, ELONGATION, and HARDNESS. The objectives were to maximize ABRASION and MODULUS. For other responses, the objective was to come close to a target value of near 500 for ELONGATION at break, and a value near 67.5 for HARDNESS. It is interesting to

see this problem as a constrained graphical optimization.

*Contour profiling*, a new feature in JMP version 3.1.5, can optimize across multiple responses graphically and interactively. After fitting the data to a response surface model, you call in the contour profiler, obtaining the controls and graphs in **Figures 1 to 3**.

Use the *control panel* to choose factors for graphing, to set current values of the factors and to set the levels of the contour lines for the responses, as shown in **Figure 1**.

The *overlay contour plot* (**Figure 2**) shows the contours for all four responses as a function of two of the three factors. As you move the slider for the response contour, the contour curves move continuously. As you move a slider for the factor values, the cross hairs move, or if the factor is not one of the axes, the contours move. You can click in the contour plot and move the crosshairs to change the current factor values.

**Figure 1. Contour Profiler Control Panel**

Check here to choose which factors to use as graph axes.

Slide this off-axis factor and the whole surface changes.

Move sliders or click and edit to change factor values. As you change factors, the current Y values change.

Red dot shows current predicted value.

Slide these or edit these to change the corresponding contour line.

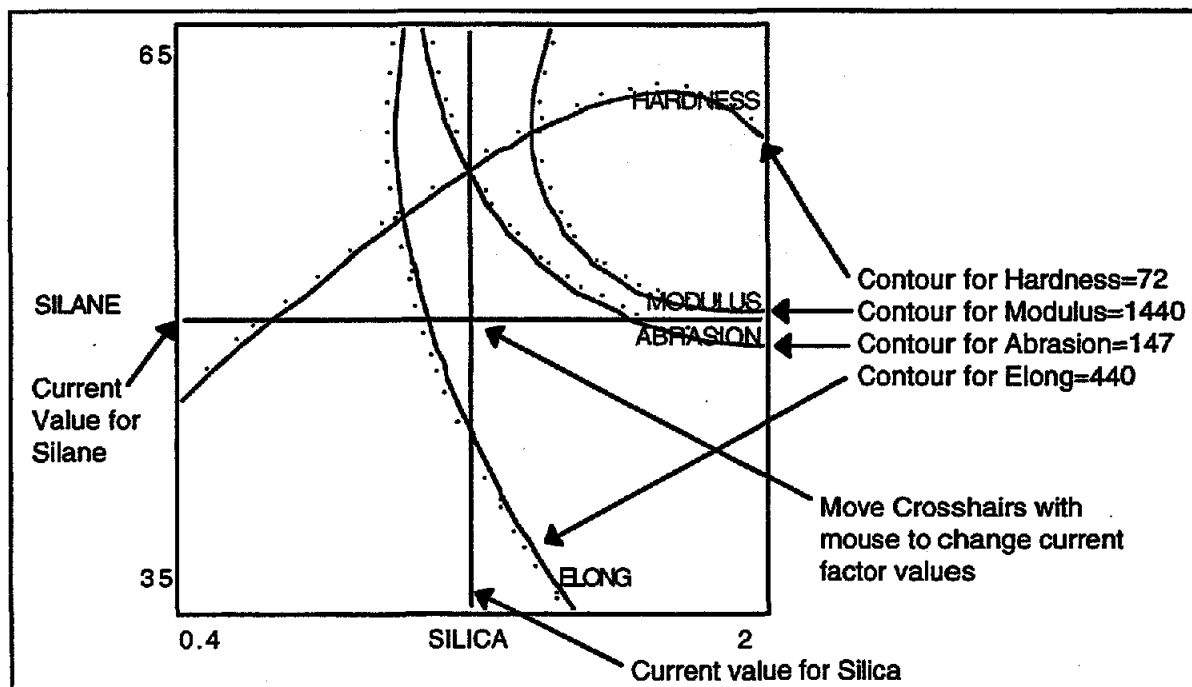
Set these to shade the area outside the limits.

| Response | Factor | Current X | Current Y | Lo Limit | Hi Limit |
|----------|--------|-----------|-----------|----------|----------|
| ABRASION | SILICA | 1.2       | 147       | •        | •        |
| MODULUS  | SILANE | 50        | 1440      | •        | •        |
| ELONG    | SULFUR | 2.2       | 440       | •        | •        |
| HARDNESS |        |           | 72        | •        | •        |

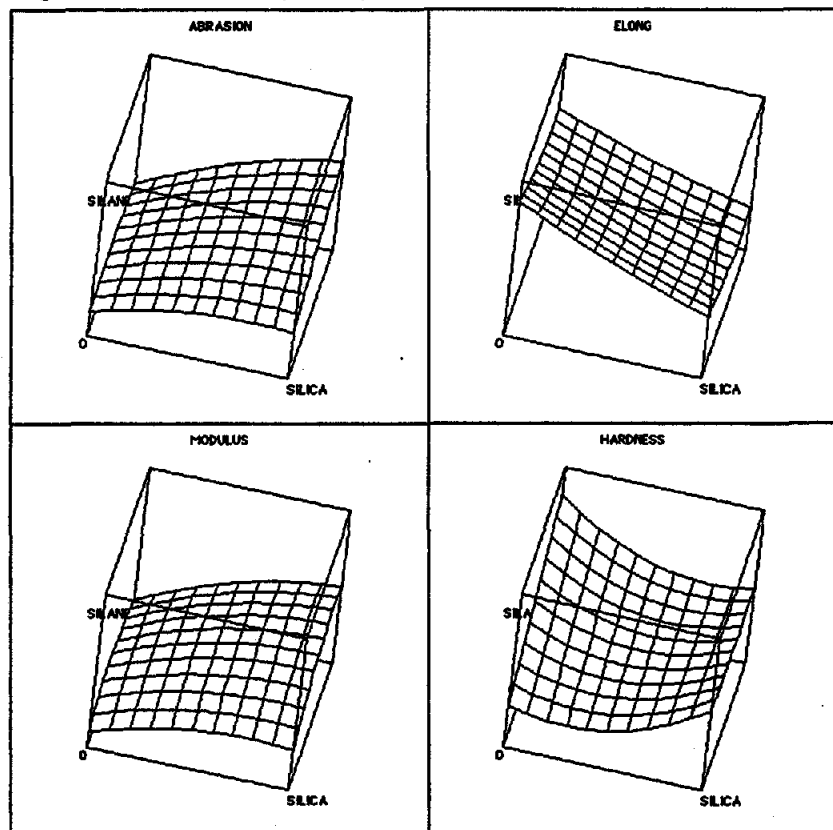
Grid Density: 20 x 20

Update Mode: Immediate

**Figure 2. Overlay Contour Plot of Current Values from Control Panel**



**Figure 3. Surface (Mesh) Plots**



The surface plots in **Figure 3** show each response surface as a function of two of the factors. The surfaces change in response to the off-axes factor changes, and contour changes. You can rotate these plots to a different viewing angle by clicking and dragging directly on them.

Now suppose you want to find the value of the three factors that:

- maximizes ABRASION, with a minimum of 120
- maximizes ELONGATION, with a minimum of 1000
- constrains MODULUS between 450 and 550
- constrains HARDNESS between 65 and 70

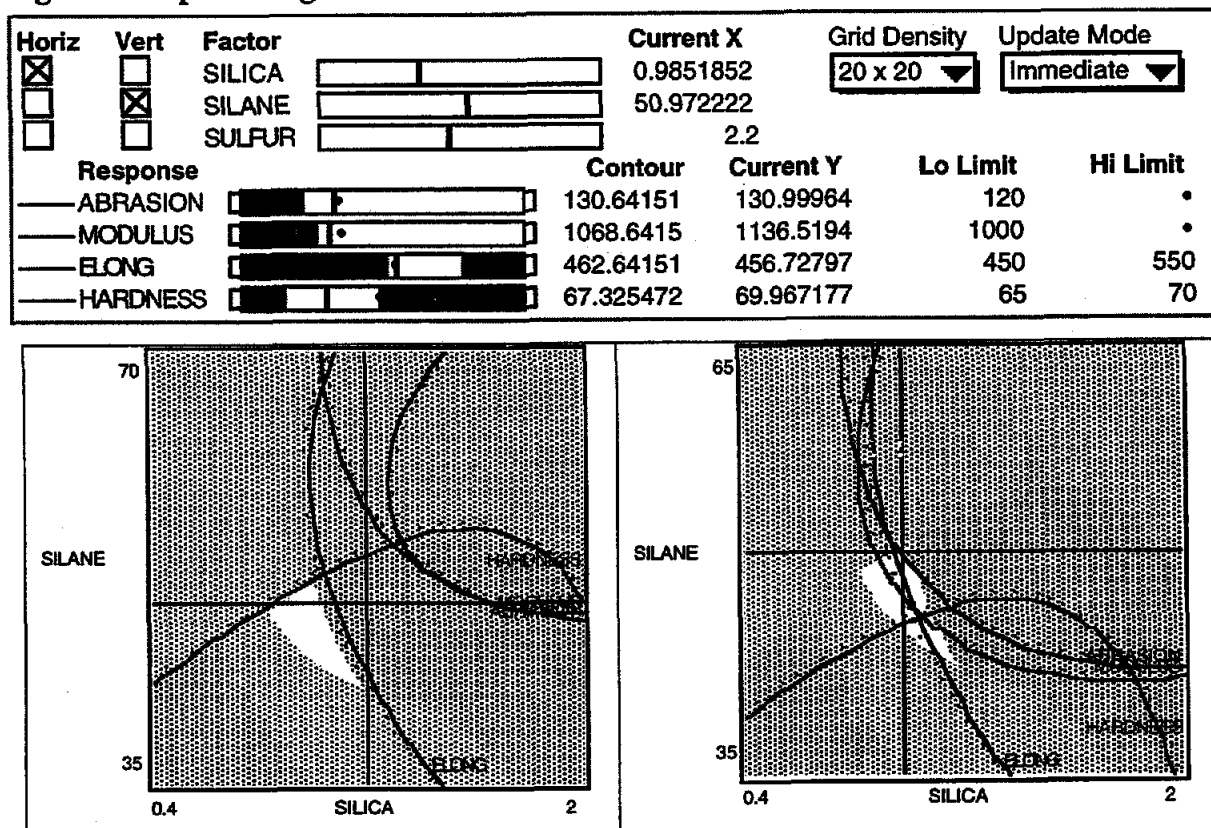
Mathematically, this is a quadratic programming problem with quadratic constraints, assuming you can specify the relative importance of the first two responses. However, you do not want a blind mathematical solution. You want to explore all the opportunities, to learn about the surfaces, about the shape of the feasible region, what trade offs are possible. So it is better to solve the problem with interactive graphics, because

you learn more, and can adapt your problem specification to the opportunities.

First enter the limit values into the limit fields. This causes the area in the contour plot to be shaded where it does not satisfy the response limits (**Figure 4**, left). The limits also show as shaded areas in the response sliders. The white area shows the feasible region that satisfies the constraints. All these plots are holding the current value of SULFUR constant; SULFUR is the factor that is not part of the plot.

Move the contours so that they are in the feasible region. You want to find the point that maximizes ABRASION and ELONGATION. The direction of greater values is shown by the side of the contour with the dots. The upper direction for the red ABRASION response, as well as the green MODULUS response is in the upper right corner of the feasible region.

**Figure 4. Optimizing the SILANE-SILICA factors.**



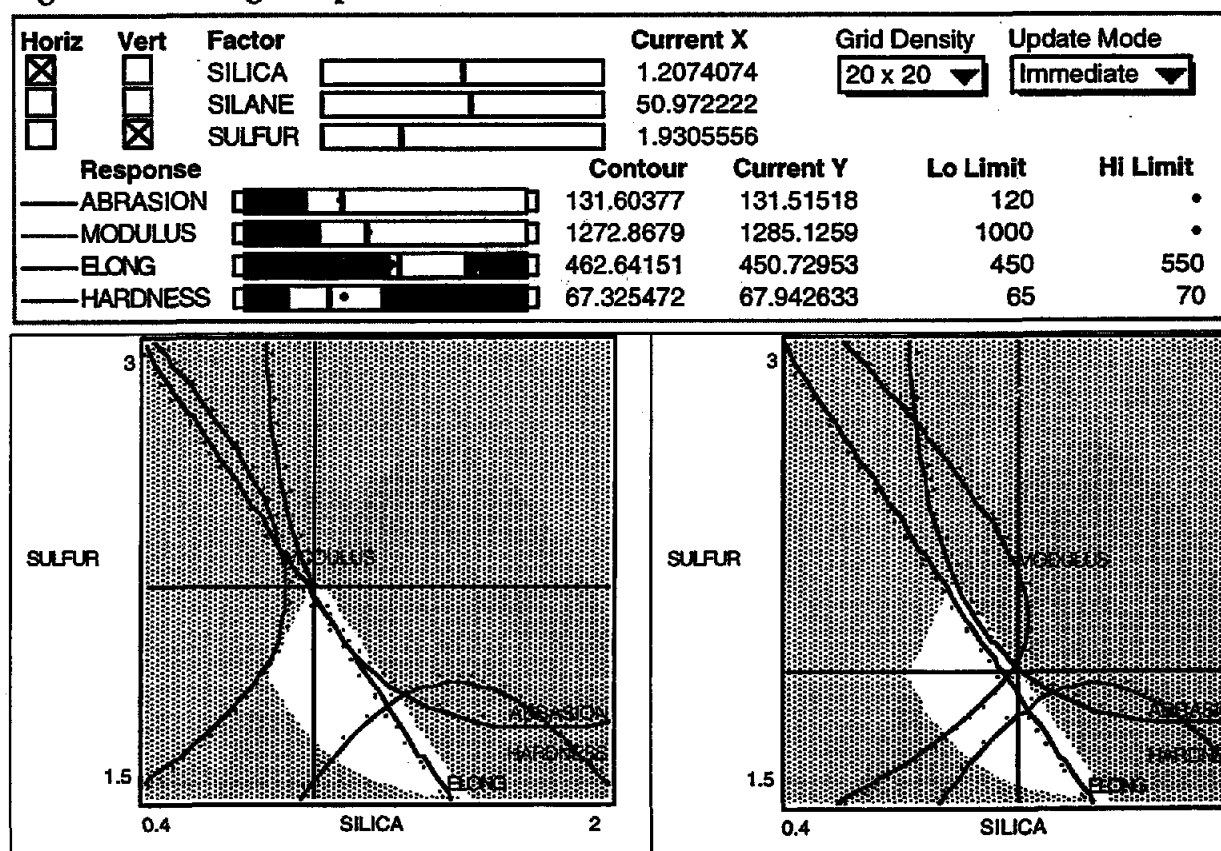
Click the plot in this corner to change the current values of SILICA and SILANE (Figure 4, Right). Moving into a corner is like a step in the simplex method for linear programming. The left plot in Figure 4 shows contours after limits are specified. The right-hand plot and the control panel are after the corner is chosen.

But this solution is only for the current value of SULFUR, the off-screen factor. Click the

Vertical checkbox to replace SILANE by SULFUR in the graph, as in Figure 5.

Following the red ABRASION contours leads to a solution partway down the right edge of the feasible region. Going farther down the edge also increases MODULUS, so we move it past the optimum for ABRASION, observing that the exact position along the edge is not very sensitive to ABRASION (Figure 5, right).

**Figure 5. Moving to Optimize SULFUR-SILICA**



Now look at the SULFUR by SILANE contours. By referring to the red ABRASION contour, and consistent with the green MODULUS contour, move into the upper right corner of the feasible area (Figure 6). This should be the optimum now.

Verify that you have achieved the optimum by checking in the other factor spaces.

Figure 7 shows a final SILANE by SILICA factor space. The feasible region has changed since Figure 4, because the value of SULFUR has now been optimized. It is in the optimum corner for ABRASION. The solution is also consistent with a high value for MODULUS.

Figure 6. Moving to Optimize SULFUR-SILANE

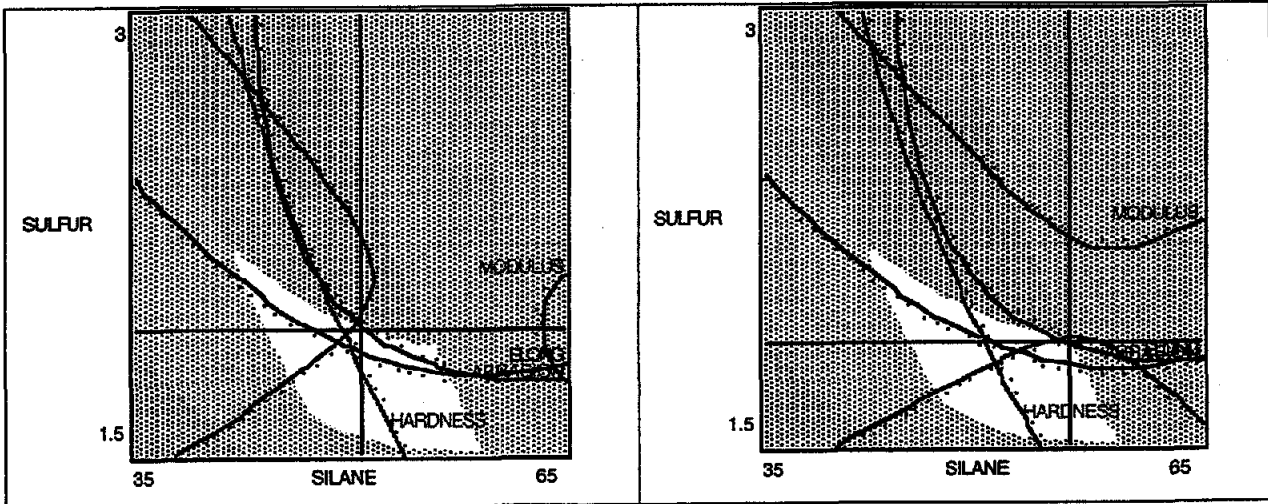
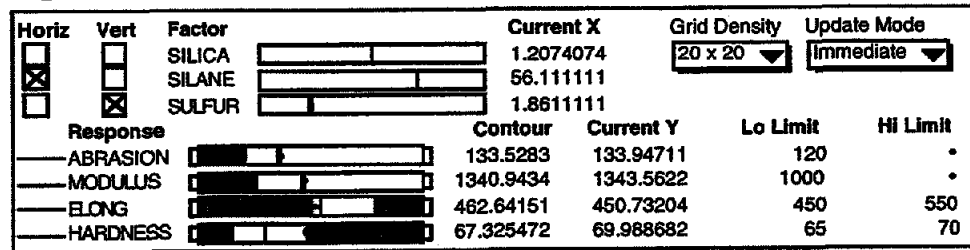
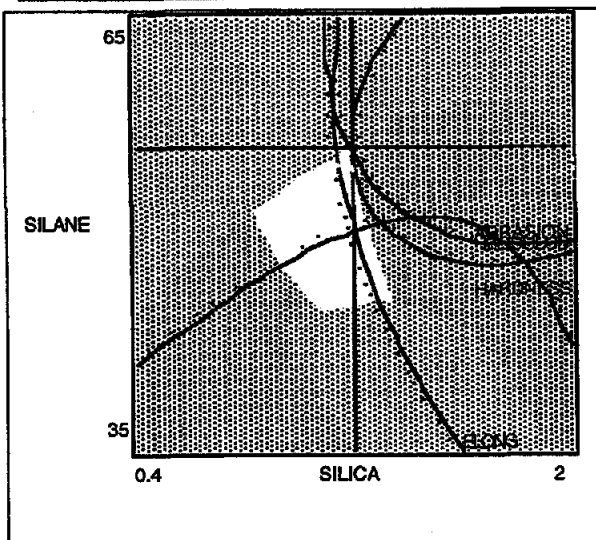
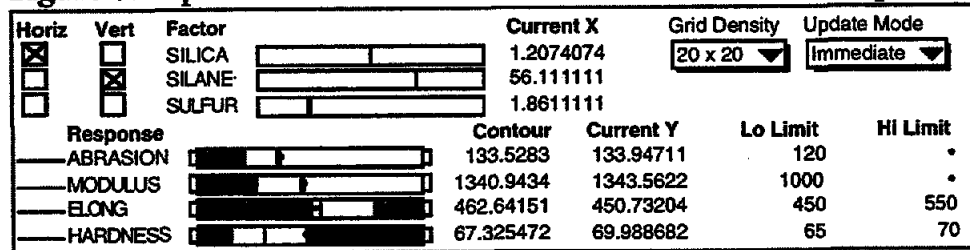


Figure 7. Optimum Now Achieved in SILANE-SILICA Factor Space



This interactive exploration of the factor space and the response surface can give insight into a complex problem by letting you explore the options and tradeoffs interactively in a few minutes, even though the problem may be complex. In this example, four quadratic response surfaces were fit to three factors. Contour profiling is one of several response surface exploration methods in JMP. Others include a canonical analysis, a response surface contour *movie*, and the *desirability* analysis pioneered by Derringer and Suich. JMP also can do the experimental design and analysis.