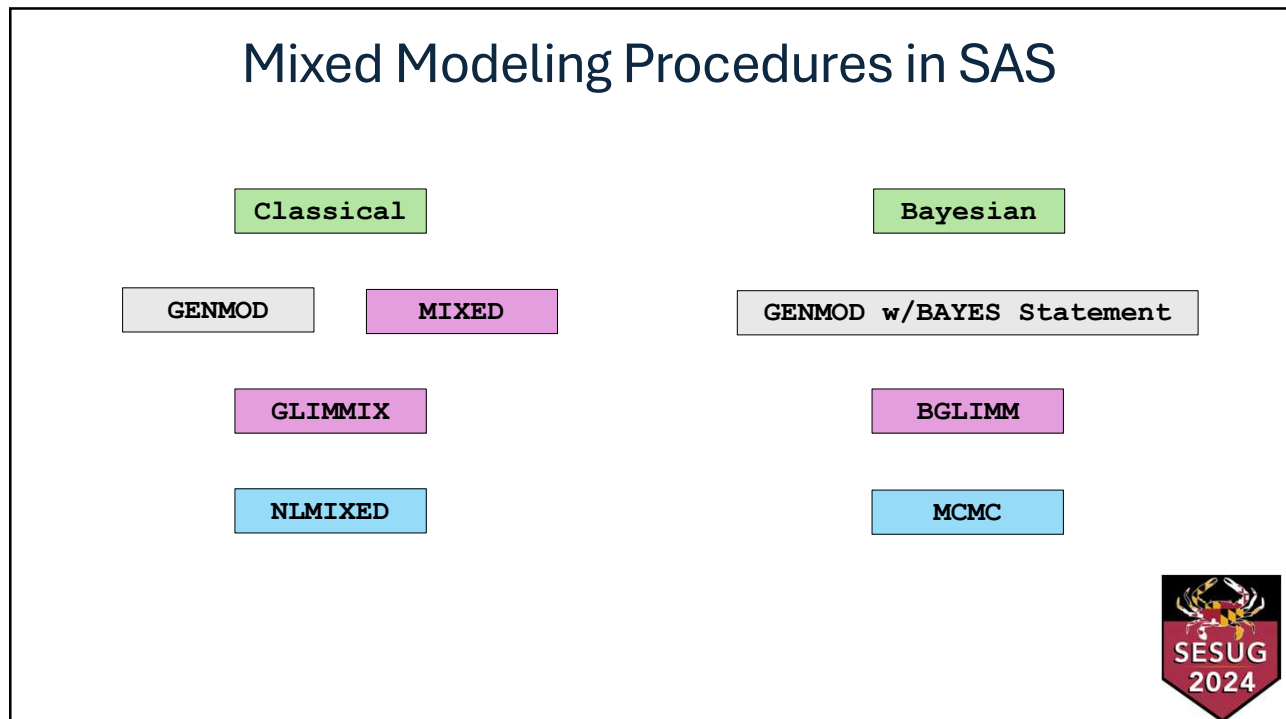




1



2

## General Linear Mixed Model

$$y = X\beta + Z\gamma + \varepsilon$$

where  $y$  is the vector of observed responses.

$X$  is the design matrix of predictor variables.

$\beta$  is the vector of regression parameters.

$Z$  is the design matrix of random variables.

$\gamma$  is the vector of random-effect parameters.

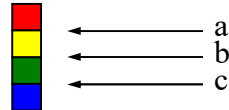
$\varepsilon$  is the vector of random errors.



3

## Toy Example

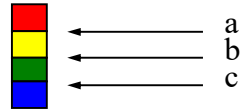
toy 1 adhesives



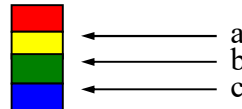
breaking strength



toy 2



toy 7



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## General Linear Mixed Model

**adhesive**  
(treatment) effects,  
fixed

**toy** (block) effects,  
random

$$y_{ij} = \mu + \beta_i + \gamma_j + \varepsilon_{ij}$$

random error

$$\gamma_j \sim N(0, \sigma_\gamma^2)$$

$$\varepsilon_{ij} \sim N(0, \sigma_T^2)$$



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## PROC MIXED Program

```
proc mixed data=sasuser.toy;
  class adhesive toy;
  model pressure=adhesive / solution ddfm=kr;
  random toy;
run;
```



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## PROC MCMC Program

```
proc mcmc data=toy seed=27513 diag=all dic outpost=mixed
  propcov=quanew thin=25 nbi=5000 ntu=5000 nmc=500000
  plots(smooth)=all mchistory=brief stats=all;
  array beta[3];
  parms beta: 0;
  parms s2t 1;
  parms s2g 1;
  prior beta: ~ normal(0, var = 1e5);
  prior s2: ~ igamma(2.001, scale = 1.001);
  random gamma ~ normal(0,var=s2g) subject=toy
    monitor=(gamma) namesuffix=position;
  mu = beta[adhesivebeta] + gamma;
  model pressure ~ normal(mu, var = s2t);
  title "Bayesian Analysis of the Toy Data Set";
run;
```



7

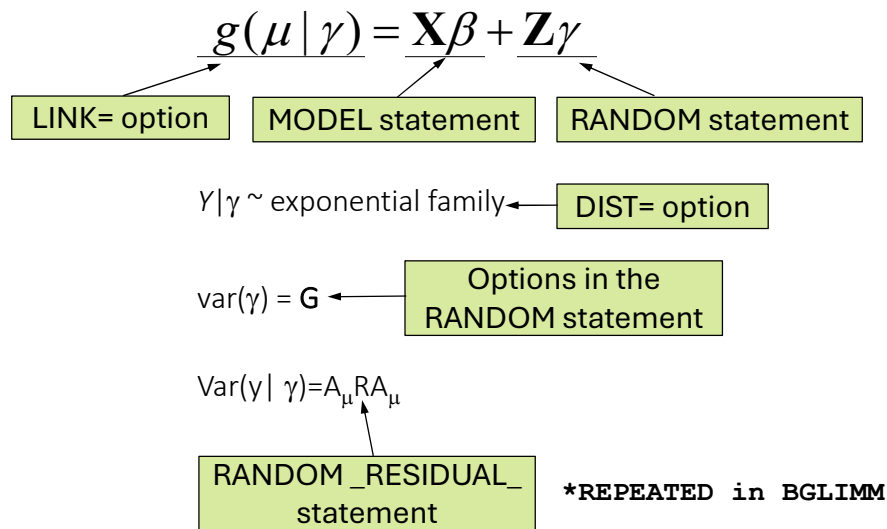
## Generalized Linear Mixed Models (GzLMMs)

- GzLMMs enable modeling random effects and correlated errors for nonnormal data.
  - A linear predictor can contain random effects.
 
$$\eta = X\beta + Z\gamma$$
  - The random effects are normally distributed.
  - The conditional mean,  $\mu|\gamma$ , relates to the linear predictor through a link function.
 
$$g(\mu|\gamma) = \eta$$
  - The conditional distribution (given  $\gamma$ ) of the data belongs to the exponential family of distributions.



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## GzLMM Formulation and PROC GLIMMIX



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## Features of BGLIMM

- Suite of covariance structures (for both G- and R-side)
- Covariance heterogeneity modeling
- Built-in priors
- Model Comparison via DIC statistic
- Multi-threading of optimal sampling



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## PROC BGLIMM Syntax

### PROC BGLIMM Statement

- DATA= names the input data set
- SEED= random seed for simulation
- OUTPOST= output a data set to contain posterior samples
- NBI= number of burn-in iterations
- NMC= number of Markov chain iterations
- NTHREADS= number of CPUs to run simulations simultaneously
- STATS= posterior statistics
- DIAG= convergence diagnostics
- PLOTS= plotting



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## PROC BGLIMM Syntax

MODEL response = fixed effects / dist= link= ...;

### • 9 response distributions:

- |                    |                     |
|--------------------|---------------------|
| • Binomial         | - Negative binomial |
| • Exponential      | - Normal            |
| • Gamma            | - Poisson           |
| • Geometric        | - Binary            |
| • Inverse Gaussian |                     |

### • 8 link functions:

- |           |                        |
|-----------|------------------------|
| • Log     | - Identity             |
| • Logit   | - Loglog               |
| • Probit  | - Complementary loglog |
| • Inverse | - PowerMinus2          |



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## PROC BGLIMM Syntax

**RANDOM** random-effects / sub= group= type= ...;

- **SUB=** option to identify the subjects for the random effects
- **GROUP=** option to identify groups by which to vary the covariance parameters; each new level of the grouping effect produces a new set of covariance parameters
- **TYPE=** option to define the covariance structure of G
  - 13 choices: AR, ARMA, CS, TOEP, UN, VC, ...

\*Multiple RANDOM statements can be used.



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## PROC BGLIMM Syntax

**REPEATED** repeated-effect / sub= group= type= ...;

- A repeated-effect is required to define the proper location of the repeated responses.
- **SUB=** option to group repeated measures together for the same subject
- **GROUP=** option to identify groups by which to vary the covariance parameters
- **TYPE=** option to define the covariance structure
  - 13 choices: AR, ARMA, CS, TOEP, UN, VC, ...



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## Sampling

- PROC BGLIMM updates parameters conditionally and sequentially through Gibbs sampling.
  - The fixed-effect parameters are drawn together first at each iteration.
  - The random-effect parameters are updated by subjects.
  - The G-side covariance parameters are then sampled.
  - Lastly, the R-side covariance parameters are updated.
- If present, missing response values are treated as parameters and are thus sampled as well.



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## Prior Distributions

- Fixed-effect parameters (Betas)
  - Flat/constant; normal
- Scale parameter
  - Inverse gamma; gamma; improper
- G-side Covariance parameters
  - Inverse wishart; inverse gamma; uniform; halfcauchy; halfnormal; siwishart
- R-side Covariance parameters
  - Inverse wishart; inverse gamma



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## Examples using BGLIMM

This demonstration illustrates the concepts discussed previously.



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Questions?

Business Card



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