

Special Control Charts for Rare Events: G-charts and T-charts

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ABSTRACT

The year of 2024 marks the 100th anniversary when Walter Shewhart introduced Statistical Quality Control Charting methods to monitor various processes such as: manufacturing operations, industrial production, hospital epidemiology, and healthcare-outcome surveillance. Numerous control charts have been developed since.

This presentation applies two special Shewhart-type (SPC) control charts, G- and T- charts, that monitors time-between-event (or rare event) situations. The example in this presentation monitors discharged patients who were hospitalized due to different COVID-19 variants of concern from March, 2021 through December, 2023.

I will review background and situations where and how these charts are used, review the basics for their construction, and analyze data using the COVID-19 hospitalization discharge example. I will produce charts using JMP® and R Integration that compare and contrast both tools.

INTRODUCTION

Walter A. Shewhart (1891-1967), a member of Bell Laboratories technical staff (1925-1956), introduced the first Statistical Quality Control chart with a memo to his boss, George Edwards, in 1924 (Figure 1). Control Charts were useful in distinguishing between assignable (special) cause and common (chance) cause sources of variation in manufacturing and producing telephones. Shewhart suggested that control charts could help reduce overall variability in production output by bringing and keeping processes subject to only chance-cause (common-cause) variation. Detecting signals and patterns on the charts revealed problems with production processes that needed to be fixed.

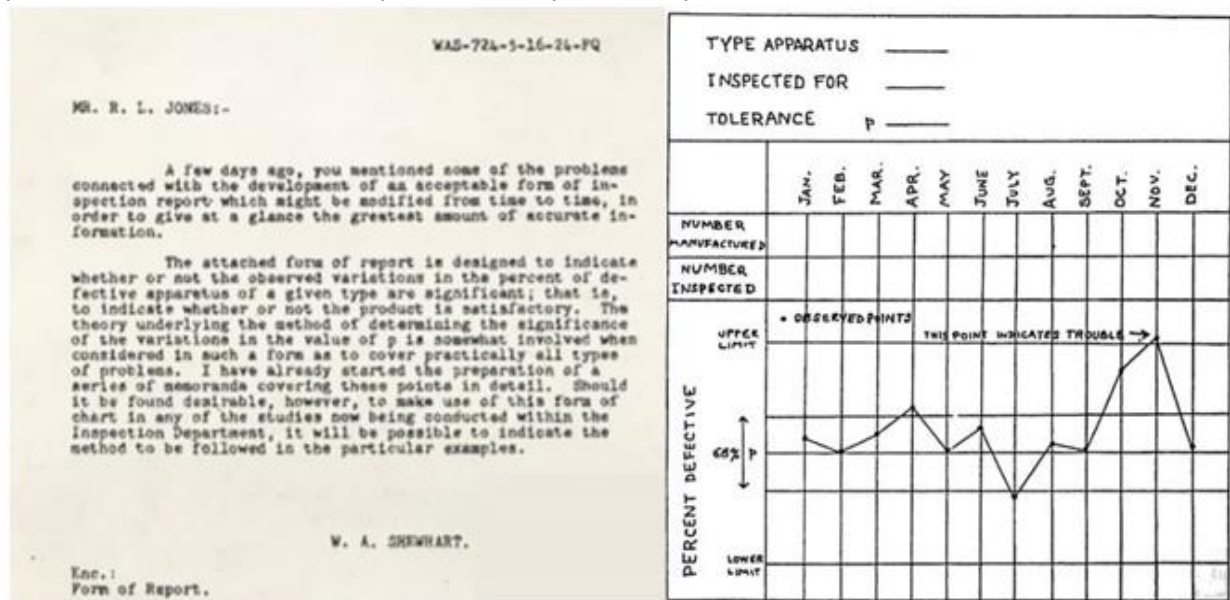


Figure 1: Walter Shewhart's May 16, 1924 memo and Control chart Example (Woodall, 2024)

By identifying and eliminating the assignable (special–source) causes, manufacturing could reduce waste and significantly improve product quality. Shewhart continued his work until he retired in 1956.

Many applications of statistical process monitoring and behavior techniques using control charts extended beyond traditional manufacturing processes. Recent examples include:

- detecting moderate-to-large shifts in parameters being monitored,
- detecting unusual patterns affecting processes,
- risk-adjusted control charts for monitoring surgical and other errors in hospitals,
- healthcare monitoring, and biosurveillance.

This presentation applies two special Shewhart-type (SPC) control charts, G- and T- charts, that monitors time-between-event (or rare event) situations.

BACKGROUND ABOUT THE G-CHART AND T-CHART

G charts (Geometric charts) are alternatives to standard attribute charts used when the events of interest are rare and discrete opportunities preceding event occurrences are counted (e.g., number of non-occurrences before the next event of interest occurs, a.k.a. “interevent count”). G -charts plot the number of opportunities of event occurrences (e.g., COVID-19 infected patients admitted in a hospital) between the events of interest (e.g., discharged). See Alexander (1994), Benneyan (2001), Kaminsky et al. (1992), and Lucas (1989).

T-charts use a Weibull power transformation for *time between rare events*. See Nelson (1994). The Weibull distribution has a shape parameter of 3.6, skewness of zero and kurtosis of 2.72. This transformation approximates the normal distribution with skewness equal to zero and a kurtosis of 3. The Weibull distribution consists of many distributional families which makes it so versatile in statistical quality and reliability applications. The Weibull transformation applied this way reduces the skewness effect.

The T is calculated by applying an Individuals control chart to transformed data ($Y = X^{(1/3.6)} = X^{0.2777}$) and back transforming the 3-sigma control limits by raising the CI to the power of 3.6 or $CI_{\text{Transformed}} = CI^{(3.6)}$.

Unlike the binomial distribution which relies on two parameters, a *fixed n* (number of trials/opportunities preceding the varying event of interest X occurs, where $X = 1$) and constant p-probability of event occurrence. The Poisson model is a limiting form of the binomial with a large n, small p with np is a moderate size. The geometric distribution relies *varying n* for the same p. The negative binomial has a fixed $X > 1$, for fixed p and varying n, where interest is on the number of preceding trials/opportunities to get the number of fixed event occurrences.

The reasons for using transformations are

- (1) to make data analysis easier to interpret, e.g., changing measurement units from inches to centimeters, once to grams, miles to meters, etc.;
- (2) stabilize the spread (variability) in data;
- (3) simplify the data structure to be more symmetric, e.g., normal or bell-shaped

Table 1 shows examples of data modelled for rare events chart that represent the times or counts between successive events. Intervals are recorded as integers or continuous values.

The geometric distribution models the non-occurrences (repeated trials or opportunities) that come before the “rare” event of interest occurs. Rare events are events which occur infrequently with low probabilities. Data for a rare events chart are often the times between consecutive events or the interval of non-event occurrences between event occurrences. Intervals can be recorded as integer counts or continuous values.

Data can be explicit counts of opportunities for occurrence that come between events, such as the number of surgeries performed between occurrences of post surgical infections, antibiotic drug resistance, such as Methicillin-resistant Staphylococcus Aureus (MRSA), Vancomycin-Resistant Enterococcus (VRE), and Vancomycin Resistant Staphylococcus Aureus (VRSA) infections.

Rare events of interest could be passes caught after pass attempts in football; the number of slot machine plays between wins; or sheets printed before ink-jet or toner cartridges are replaced or copier maintenance is performed. Each event of interest is preceded by the number of non-occurrences of the event (a.k.a., the *Inter-event count*). The time between when events occur, T-chart, follows the geometric distribution, too.

Rare Event	Inter-event Counts/Opportunities/Trials before or until the Event Occurs	Event of Interest
Sports (Football/Soccer, Basketball/Baseball)	Number of attempted field goals, passes, shots, or pitches thrown	Number of completed passes, scored field goals, free throws made, base hits or home runs
Slots in casinos	Number of games played	Number of wins (or losses)
Hospital Epidemiology (study, identification, and preventive ways to avoid adverse health outcomes within hospitals)	Number of admitted, infected, or treated patients	Number of COVID-19 or nosocomial (hospital-acquired) infected patients who were discharged or experienced harmful medication errors (e.g., wrong site on patient surgeries) during hospitalization
Copier, laser/ink-jet printers	Number of copied or printed sheets	Number of paper jams, maintenance calls, toner/ink-jet cartridge replacements
Automobile, medical or electronic devices manufacturing	Number of cars, hardware units produced	Occurrence of failures, line-stoppages, service breakdowns, recalls, etc.
Service	Number of Bank Transactions	Number of Bank Transaction Errors
Education	Number of operational days	Number of school closings, days of absences or tardiness by students/teachers/aides/staff

Table 1: Examples of “Rare Event” Situations

Rare events charts gained increased acceptance in health care quality improvement applications because they are easy to use and are suitable for processes that have low defect rates. Standard Shewhart control charts, like the c- or u-charts, tend to give more, false out-of-control (OCC) signals that result in very narrow control limits. Therefore, the c charts and other traditional control charts will not perform so well when used to monitor rare events.

G-charts are effective in understanding whether rare events occur more frequently than expected and signal when to intervene corrective actions. G-charts count the number of possible opportunities since the

last event. If you plot this type of data using a standard Shewhart control chart, you might see many more false signals, as the limits might be too narrow.

T-charts are extensions of the g-chart, which typically plots the number of opportunities between events that monitor the amount of time between events. Like the g-chart, the t-chart is used to detect changes in the rate at which the event of interest occurs. Points above the upper control limit indicate how much time between event occurrences has increased. Thus, the rate of the events has decreased. Points below the lower control limit indicate that the rate of events of interest has increased. A t-chart can be used for numeric, nonnegative data, date/time data, and time-between data:

- Numeric, nonnegative data is the number of intervals between events. It can be continuous or integer.
- Date/time data records the date and time of each event. Each data value must be greater than or equal to the preceding value.
- Time-between data (also known as elapsed-time data) represent the elapsed time between event i and event $i-1$. If there are no 0's in the data, then estimates of the shape and scale parameters (skewness and kurtosis) are calculated from the data and used to obtain the percentiles of the Weibull distribution.

STEPS IN GENERATING G- AND T- CHARTS

1. Create vector of random values from a geometric distribution.
2. Define a baseline of $N = 31$ days when patients were discharged. Let M = the number of patients discharged until the k th ("successful" event) COVID-19 infected patients discharged (where $k-1$ patients are discharged without COVID-19 infections that were non-occurrences between successes). Suppose the probability of occurrence is: $p = 0.107$.
3. The probability model, $p_M(k)$, is equal to: $p_M(k) = (1 - 0.107)^{(k-1)} * (0.107)$, for $k=1,2,3,..., 31$.
4. The expected value, $E(M) = 1/p = 1/0.107 = 9.35$, where $p = 0.107$.
5. $Var(M) = (1-p)/p^2 = (1-0.107)/0.107^2 = 77.998$

The 3-sigma limits for the G-chart is calculated by $\bar{x} \pm 3 * \sqrt{\bar{x} * \bar{x} + 1}$, where $\bar{x}$ = average number of opportunities between events. Other software use 95% probability limits (or 2-sigma warning limits).

If we do not have information on the opportunities (treated COVID-19 infected patients), we may instead use a T chart for time between events (number of days between discharges)

The 3-sigma limits for the T-chart for time between rare events may be calculated by applying an Individuals (X) control chart to transformed data ($y = X^{(1/3.6)} = X^{0.2777}$) and back transforming the 3-sigma limits (i.e., $CL = CL^{3.6}$).

The Weibull power transformation, $y = X^{(1/3.6)}$, transforms X to approximate a normal distribution with a skewness (departure from symmetry) = 0, and kurtosis (distribution shape, peaked or flat) = 3 based on Nelson (1994)'s article.

Table 2 shows the Control Limits for the G- and T-charts.

	Nonconforming rate (p) known	Nonconforming rate estimated ($\bar{X}$)
Upper control limit (UCL g)	$r \cdot \frac{1-p}{p} + k \cdot \sqrt{\frac{1-p}{p^2} \cdot r}$	$r \cdot \bar{X} + k \cdot \sqrt{\bar{X} \cdot (\bar{X} + 1) \cdot r}$
Center Line (CL)	$r \cdot \frac{1-p}{p}$	$r \cdot \bar{X}$
Lower control limit (LCL g)	$r \cdot \frac{1-p}{p} - k \cdot \sqrt{\frac{1-p}{p^2} \cdot r}$	$r \cdot \bar{X} - k \cdot \sqrt{\bar{X} \cdot (\bar{X} + 1) \cdot r}$

Table 2: Center Line and Control Limits for g- and T- control charts

where $r = 1$ (geometric distributed), or $r > 1$ (negative binomial distributed) cases.

Note: Some software programs, like Minitab, the RAREEVENTS procedure in SAS/QC®, and R (R Core Team, 2022), may compute the CL = $0.693 (r \cdot \bar{X})$ to reduce any large skewness effects associated with the geometric distribution. This calculation converts the center line into the theoretical median (middle value). The median acts as a substitute for the mean ($\bar{X}$), that is less sensitive to extreme (skewed) values in rank-ordered data.

CASE STUDY EXAMPLE

This example uses data of from an un-named, neighborhood hospital where patients were admitted and treated for COVID-19 infections during March, 2021-December, 2023.

Since the hospital had a limited number of beds and ventilators, hospital administration officials needed to manage the influx of admissions by COVID-19 infected patients and minimize the lengths of stay (LOS), discharging them as soon as possible.

They chose to use G- and T- controls charts to monitor intervals or time between the rare outcomes of COVID-19 discharges.

We use a G chart for opportunities, trials, or non-event occurrences (e.g., hospital patients admitted with COVID-19 infections) between events (discharges).

G-chart data tracks the units produced between events, e.g., asymptomatic COVID-19 infected patients discharged from the hospital with a discharge probability of $p = 0.107$ (rounded to 0.11 or 11%)

If events are rare and the subgroups are small, then Shewhart C, U, and P charts become useless because we follow the number of independent trials (n) with a success probability of p before the single *specific* event of interest (with a geometric probability distribution) or *fixed* number of the *specific* event of interest (modelled by negative binomial or Weibull distributions) occurs.

Say, for example, 11% of discharged patients were treated for COVID-19 infections with the average weekly number of discharges in a small hospital department is 10.76 (~11), the hospital would, on average, expect to have less than one COVID-19 case per week.

Instead, we could plot the number of discharges between each discharged patient who were treated with or without COVID-19. The number of units between events is modelled by the geometric distribution. The G-chart (or Geometric chart) is an alternative to a standard attribute chart when events of interest are rare and discrete opportunities between events are counted (e.g., number of units or days between event occurrences).

The generalization of geometric distribution is the *negative binomial* which counts the number of trials between a fixed number of r (>1) events occurs. Negative binomial distribution-based control charts are called H-charts. Throughout this presentation the G-chart will be used to represent both geometric and negative binomial distributions.

The geometric distribution has two definitions:

- 1) Count the number of "non-event" occurrences until the first (or more than one) "event" happens (e.g., discharged COVID-19 infected patient) occurs, $X = 0, 1, 2, \dots$
- 2) Number of trials until the first (or more than one) "event" occurs (e.g., discharged COVID-19 infected patient) where $X = 1, 2, 3, \dots$, etc.

Table 3 shows the rare events used to plot the Covid19 discharged patients occurring between days during the months of March 2, 2021 through December 25, 2023. Individual discharges are identified in order of occurrence on the Discharge dates on the horizontal axis. For each event, the time in days since the previous event is reported. The rare events (g-) chart is better suited than traditional control charts to detecting changes in the frequency of low-probability events. The Dominant Variant column denotes the COVID-19 predominant SARS-CoV-2 strains that were active during the time period of patient hospitalization.

Date	covid19_data	days_between_discharge	Dominant_Variant	Date	covid19_data	days_between_discharge	Dominant_Variant	Date	covid19_data	days_between_discharge	Dominant_Variant
03/02/2021	3	• Alpha, Beta		07/02/2021	79	2 Delta		12/14/2021	7	18 Omicron	
03/10/2021	10	8 Alpha, Beta		07/05/2021	2	3 Delta		12/16/2021	0	2 Omicron	
03/27/2021	25	17 Alpha, Beta		07/24/2021	5	19 Delta		12/20/2021	16	4 Omicron	
03/30/2021	12	3 Alpha, Beta		07/28/2021	38	4 Delta		12/22/2021	1	2 Omicron	
03/31/2021	3	1 Alpha, Beta		07/31/2021	29	3 Delta		12/23/2021	2	1 Omicron	
04/02/2021	30	2 Alpha, Beta		08/07/2021	2	7 Delta		01/02/2022	2	10 Omicron	
04/13/2021	11	11 Alpha, Beta		08/10/2021	12	3 Delta		01/17/2022	4	15 Omicron	
04/20/2021	11	7 Alpha, Beta		08/12/2021	0	2 Delta		02/01/2022	1	15 Omicron	
04/30/2021	12	10 Alpha, Beta		08/13/2021	1	1 Delta		02/02/2022	14	1 Omicron	
05/02/2021	6	2 Alpha, Beta		08/22/2021	32	9 Delta		02/20/2022	14	18 Omicron	
05/04/2021	0	2 Alpha, Beta		08/30/2021	0	8 Delta		03/21/2022	10	29 Omicron	
05/11/2021	11	7 Alpha, Beta		09/17/2021	18	18 Delta		06/01/2022	10	72 Omicron	
05/16/2021	8	5 Alpha, Beta		09/26/2021	24	9 Delta		06/10/2022	1	9 Omicron	
05/17/2021	13	1 Alpha, Beta		10/08/2021	4	12 Delta		07/09/2022	25	29 Omicron	
05/23/2021	9	6 Alpha, Beta		10/12/2021	6	4 Delta		07/26/2022	1	17 Omicron	
05/28/2021	0	5 Alpha, Beta		10/17/2021	3	5 Delta		07/30/2022	3	4 Omicron	
05/30/2021	18	2 Alpha, Beta		10/22/2021	8	5 Delta		08/01/2022	19	2 Omicron	
06/03/2021	14	4 Alpha, Beta		10/28/2021	4	6 Delta		08/07/2022	9	6 Omicron	
06/07/2021	7	4 Alpha, Beta		10/31/2021	22	3 Delta		08/21/2022	21	14 Omicron	
06/11/2021	3	4 Alpha, Beta		11/02/2021	1	2 Delta		09/04/2022	1	14 Omicron	
06/15/2021	34	4 Alpha, Beta		11/12/2021	2	10 Delta		09/12/2022	5	8 Omicron	
06/17/2021	2	2 Alpha, Beta		11/19/2021	2	7 Delta		09/16/2022	2	4 Omicron	
06/21/2021	32	4 Alpha, Beta		11/20/2021	6	1 Delta		10/27/2022	3	41 Omicron	
06/24/2021	11	3 Alpha, Beta		11/24/2021	0	4 Delta		11/01/2022	11	5 Omicron	
06/29/2021	29	5 Alpha, Beta		11/26/2021	11	2 Delta		12/14/2022	15	43 Omicron	
06/30/2021	33	1 Alpha, Beta						12/24/2022	20	10 Omicron	
								01/01/2023	13	8 Omicron	

Date	covid19_data	days_between_discharge	Dominant_Variant
02/20/2023	9	50	Eris
03/07/2023	6	15	Eris
03/14/2023	1	7	Eris
04/04/2023	2	21	Eris
04/14/2023	19	10	Eris
05/25/2023	11	41	Eris
07/22/2023	5	58	Eris
09/03/2023	14	43	Pirola
09/12/2023	0	9	Pirola
09/22/2023	2	10	Pirola
09/25/2023	8	3	Pirola
10/28/2023	21	33	Pirola
11/22/2023	6	25	Pirola
12/25/2023	9	33	Pirola

Table 3: COVID-19 Discharge Data (faceted by Dominant Variants)

These data shows that the rare events used to plot the Covid19 discharged patients occurring between days during the months of March 2, 2021 through December 25, 2023. Individual discharges are identified in order of occurrence on the Discharge dates on the horizontal axis. For each event, the time in days since the previous event is reported. The rare events (g-) chart is better suited than traditional control charts to detecting changes in the frequency of low-probability events.

Figure 2 G-Chart from R shows discharges fell within the control limits, (except for 79 discharges on 7/02/2021). Note: Upper Control Limits uses 2-sigma probability limits instead of 3-sigma Shewhart Control Limits. See Anhoej (2021).

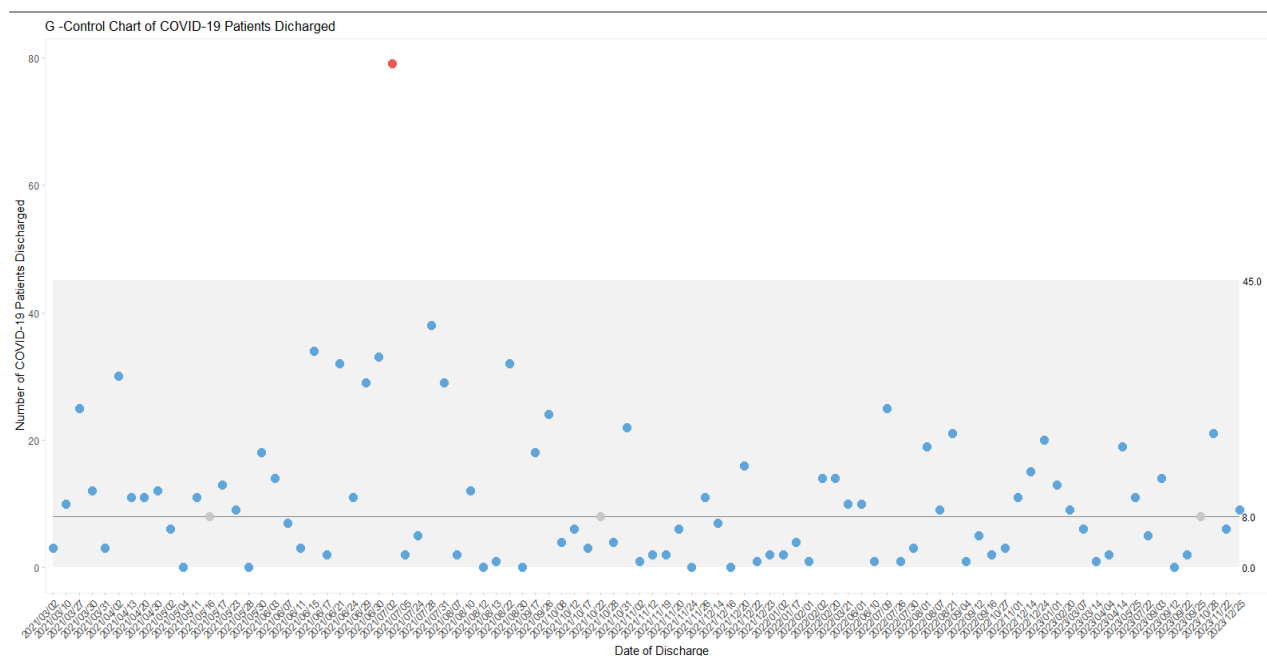


Figure 2: G-Chart of COVID-19 discharges from R

LCL = 0, CL= 8, UCL = 44.162

The G-Chart from R shows discharges fell within the control limits, (except for 79 discharges on 7/02/2021). Note: Upper Control Limits uses 2-sigma probability limits instead of 3-sigma Shewhart Control Limits.

Figure 3: G-Chart from JMP®'s shows the number of COVID19 discharged patients fell within the control limits, (except for 79 discharges on 07/02/2021), like the discharges in Figure 2.

LCL = 0, CL= 8, UCL = 44.9888

G-Chart from JMP®'s shows the number of COVID19 discharged patients fell within the control limits, (except for 79 discharges on 07/02/2021)

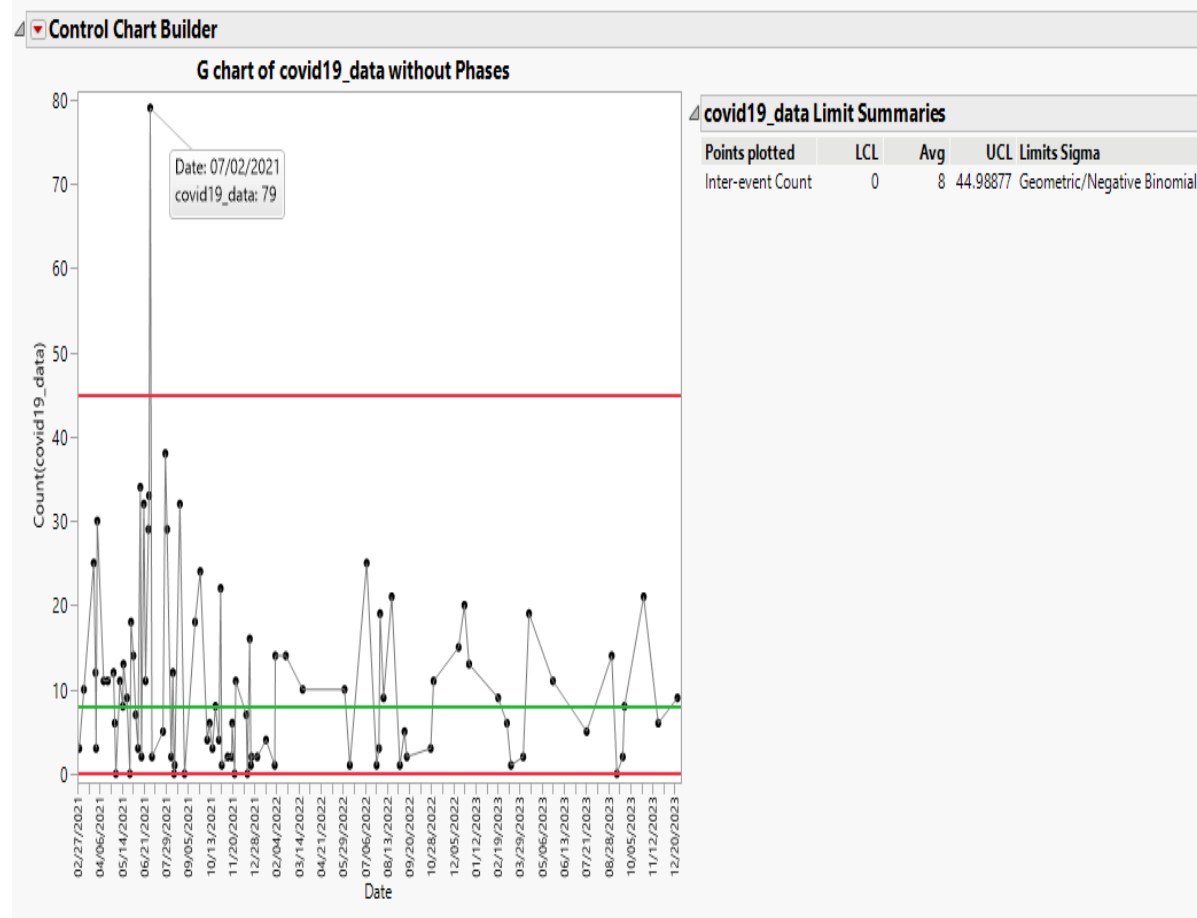


Figure 3: G-Chart of COVID-19 discharges from JMP®

Figure 4 is the T-Chart from R showing the longest time between discharges were approximately 72 and 58 days on 6/01/2022 and 7/22/2023, respectively.

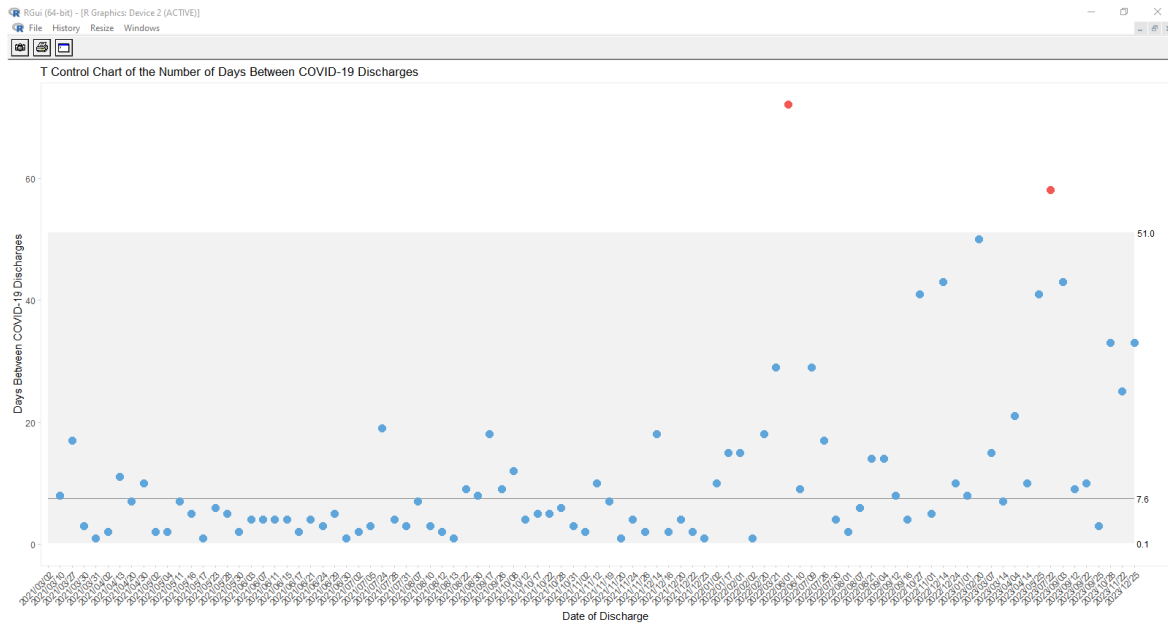


Figure 4: T-Chart of Time Between COVID-19 Discharges from R

LCL = 0.09, CL= 7.55, UCL = 51.05

The T-Chart from R shows the longest time between discharges were approximately 72 and 58 days on 6/01/2022 and 7/22/2023, respectively.

Figure 5 is the T-Chart from JMP®'s Control Chart Builder showing the same times between discharges from the previous T-chart produced by R (72 and 58 days on 6/01/2022 and 7/22/2023, respectively).

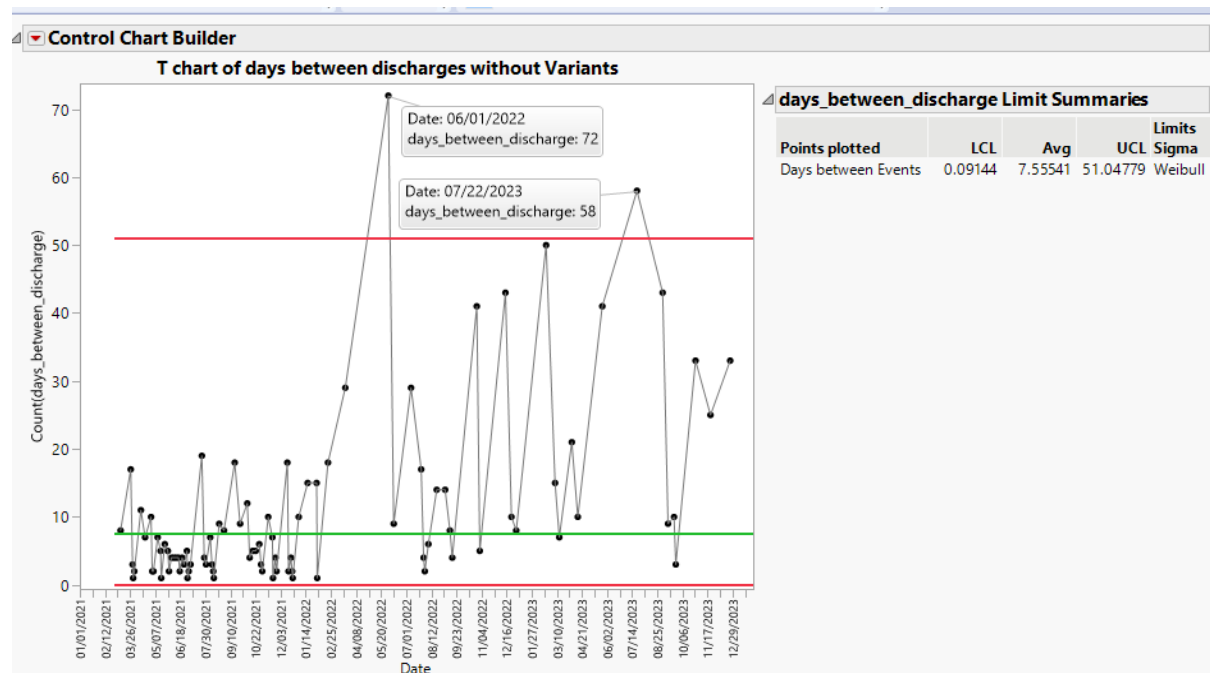


Figure 5: T-Chart from JMP®'s Control Chart Builder showing the same times between discharges from Figure 4

LCL = 0.09144, CL= 7.55541, UCL = 51.04779

T-Chart from Figure 5 shows the same times between discharges from the previous T-chart produced by R (72 and 58 days on 6/01/2022 and 7/22/2023, respectively).

Figure 6 shows the G-chart of discharge patients, taking the Dominant Variants into account. Most of the discharges fell within the control limits, except for the 79 discharges with the Delta Variant on 7/2/2021.

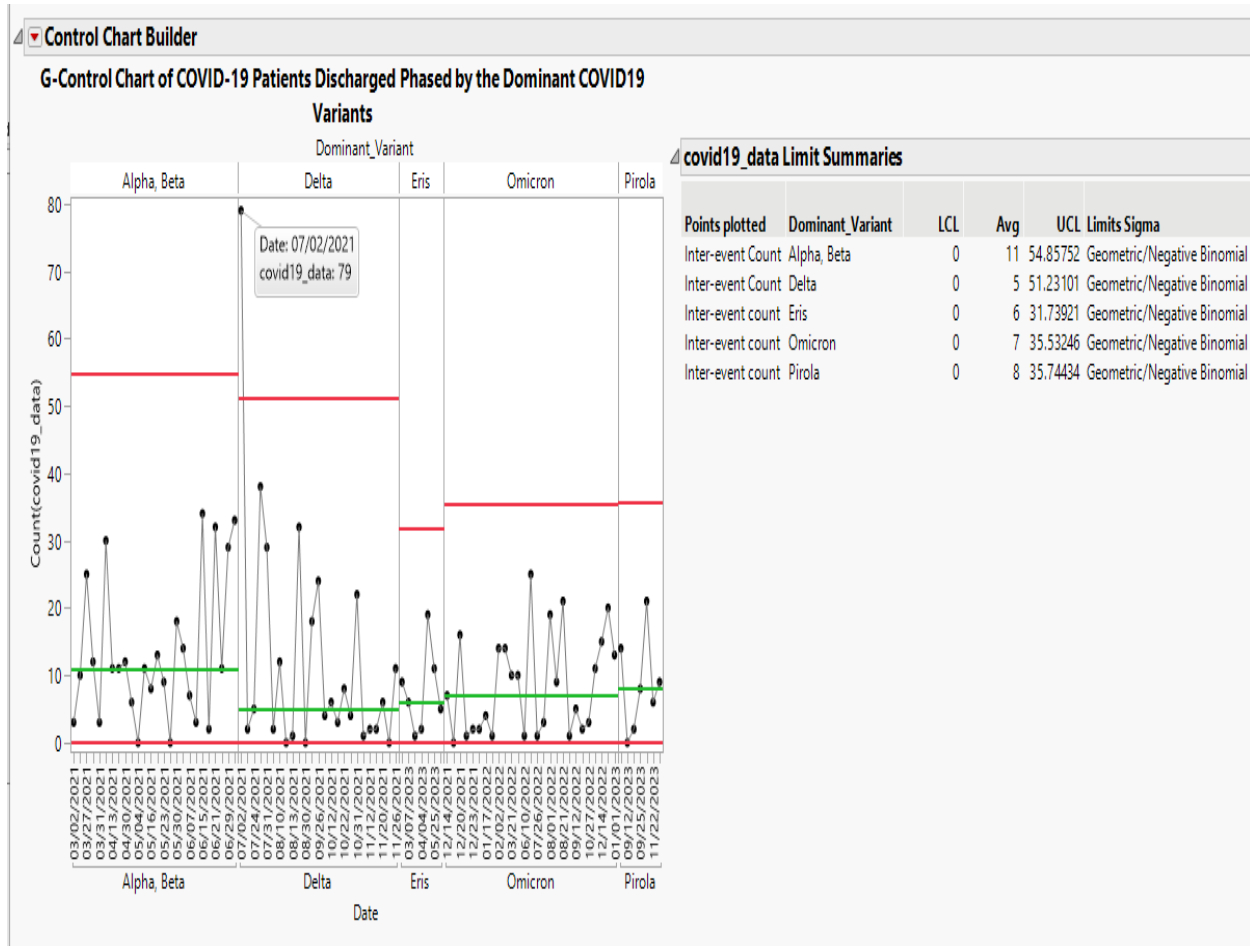


Figure 6: G-Charts of COVID-19 Discharges Phased by Dominant Variants from JMP®'s Control Chart Graph Builder

Figure 6 showed lower variation in the spread of discharges during the Eris variant period from February 20, 2023 through May 25, 2023. Most of the discharges fell within the control limits, except for the 79 discharges with the Delta Variant on 7/2/2021

Also, the chart showed lower variation in the spread of discharges during the Eris variant period from February 20, 2023 through May 25, 2023.

Figure 7 displays the time order sequence of discharges separated by COVID-19 variants form R.

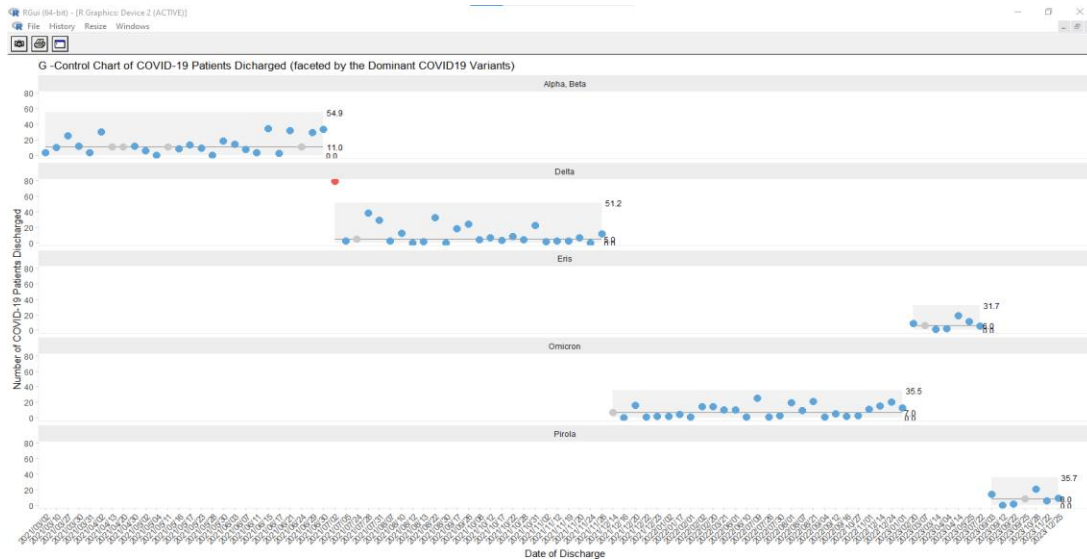


Figure 7: G-chart of COVID-19 Discharges Phased by Dominant Variants from from R's qicchart2 package

Figure 8: shows that JMP®'s Control Chart Builder plot of the Time between Discharges fell within the T-chart control limits.

Any points outside the control limits would lead to the search for assignable causes to explain what is going on that would help manage bed and ventilator availability when treating patients in the future.

Figure 8 is the T-Charts from JMP®'s Control Chart Builder plot of the Time between Discharges, phased by Dominant Variants.

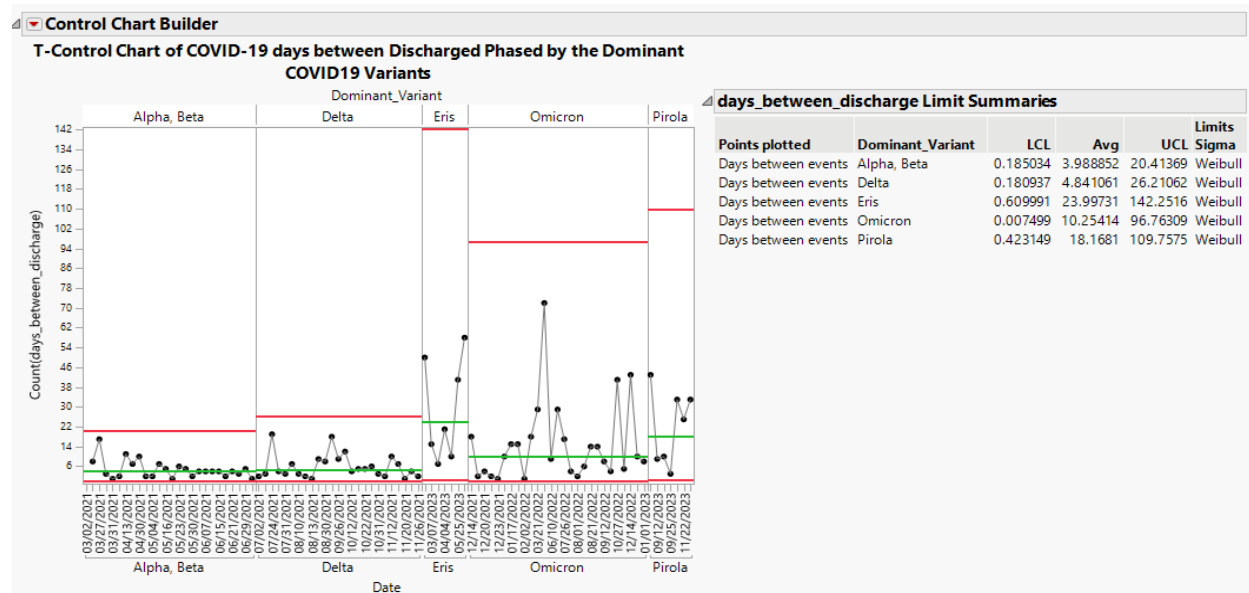


Figure 8: T-Charts from JMP®'s Control Chart Builder plot of the Time between COVID-19 Discharges Phased by Dominant Variants

The Time between Discharges fell within the T-chart control limits.
Any points outside the control limits would lead to the search for assignable causes to explain what is going on that would help manage bed and ventilator availability when treating patients in the future.

Figure 9 shows the same T-chart from R that was shown in Figure 8

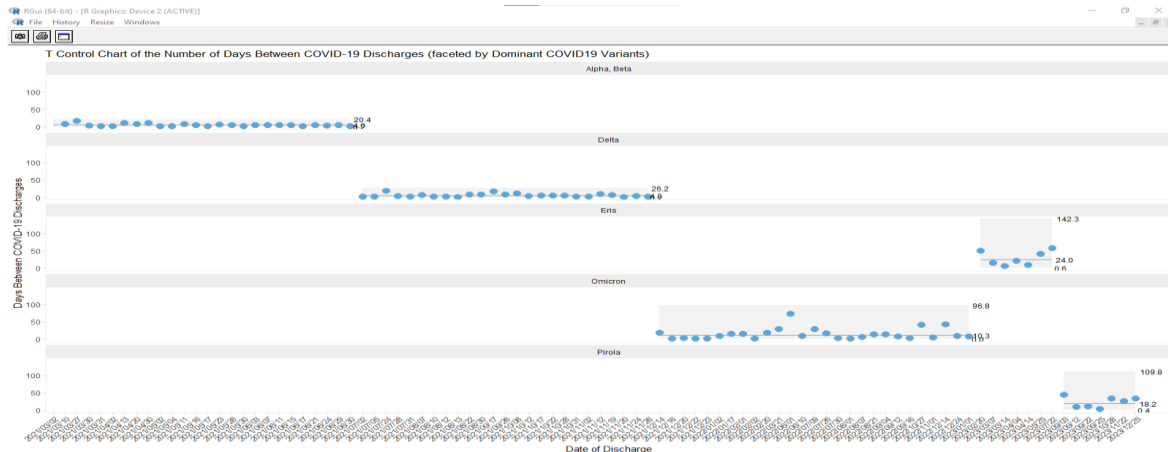


Figure 9: T-chart of the Time between COVID-19 Discharges Phased by Dominant Variants from R's qicchart2 package

Figure 10 shows departures of fitted geometric and negative binomial models. This suggests that the dominant variants could account for some of the model fitting differences in the number of discharged patient densities. Judging by the QQ-Plot, the geometric model fits a bit better up to the 30th theoretical quantile. Correspondingly, the AIC of the geometric fit is lower (633.4096) compared with the negative binomial fit (635.3899), disregarding the effects of the dominant variants. Models with the lowest AIC value are considered to be more accurate among other compared models.

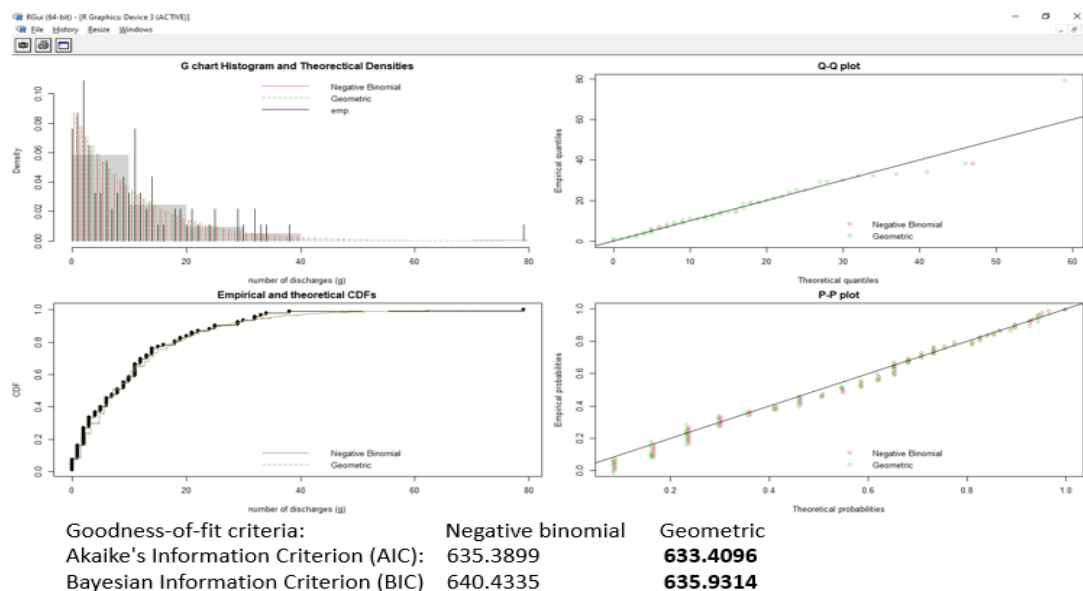


Figure 10: G-chart Goodness-of-fit plots (density, CDF, Q-Q, and P-P) on the fitted data with legends of the Negative Binomial, Geometric distributional fits to the number of discharges

Figure 11 shows departures of the fitted Weibull, Gamma, and Log-normal models. This suggests that the dominant variants may account for the model fitting differences in days between discharges. The AIC overall was lower with the lognormal fit than the Weibull or gamma fits (Lognormal AIC = 611.8302, Weibull AIC = 626.9351, gamma AIC = 635.1386, respectively). All models look good as shown on the QQ-Plot up to the 20th theoretical quantile before they depart more extremely afterwards.

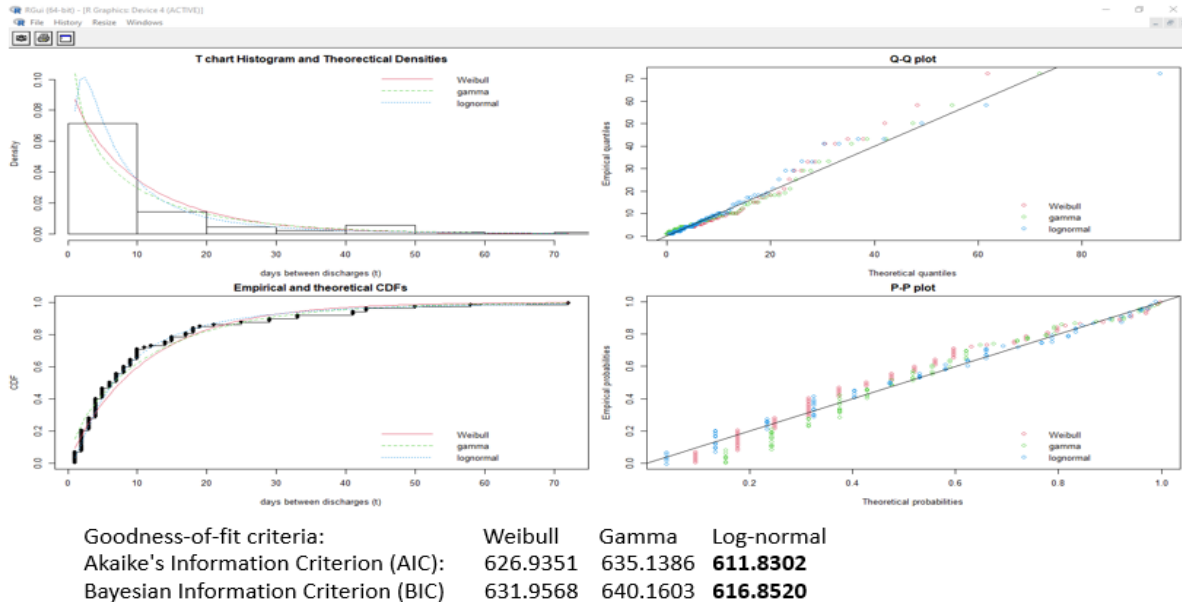


Figure 11: T-chart Goodness-of-fit plots (density, CDF, Q-Q, and P-P) on the fitted data with legends of the Weibull, Gamma, and Lognormal distributional fits to the days between discharges

SUMMARY AND CONCLUSIONS

The Geometric (g-) control chart described in this presentation apply to single events. Typical count or attributes control charts have a varying numerator (what is counted) and fixed denominator (the sample size or items in the pool to count).

G-charts have varying denominator with a fixed numerator. If many periods have long periods of zero events (when no incidents occurred), a chart may show many flat lines plotted along the x-axis. When an event occurs, the counted event for the period is plotted and the inter-event count resets to zero until the next event occurs.

As this cycle continues the resulting control chart may look odd rendering misleading signals of improved quality performance if the events of interest are nonconformances between conformances. Therefore, it is important to “operationally define” what the event of interest is before charting begins.

Operational definitions consist of:

- *criterion* or *criteria* for judgment,
- *specific tests* or *measures of items or units* coming from some repeatable process,
- a *decision rule* that tells whether units/items meet the criteria or not, Deming, (1982).

Multivariate geometric extensions exist. Derivations are made for waiting-time outcomes from multinomial using the Marshall and Olkin (1967) multivariate exponential distribution approximation to multivariate geometric (beyond the scope of this presentation).

Esary and Marshall (1973) and Mai, Scherer, and Shenkman (2013) provide more information about this approach.

Shewhart's ideas have extended to the big data, Artificial Intelligence (AI), machine learning, cyber-physical systems (CPS), the Internet of Things (IoT), cloud computing, and smart-device areas that characterizes Industry and Quality 4.0. Recent applications of control charts include: monitoring social network data involving large sparse square matrices containing attribute data like the number of individuals in networks; continuous data collected from environmental-monitoring sensors; public health and financial-surveillance; or coordinate-measurement-machines (CMM), computer-numerical-control (CNC) systems, and 3D printing technologies.

What Shewhart said in 1939 (p. 49) still pertains in 2024: "The long-range contribution of statistics depends not so much on getting a lot of highly trained statisticians into industry as it does on creating a statistically minded generation of physicists, chemists, engineers, and others who will in any way have a hand in developing and directing the production process of tomorrow."

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